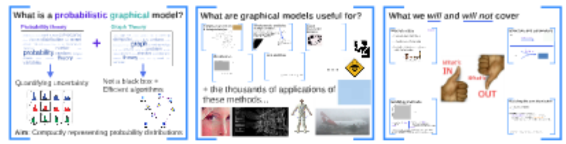
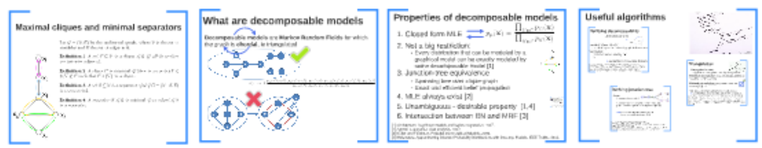
Scalable learning of graphical models

Introduction - Motivation



Graphical models 101

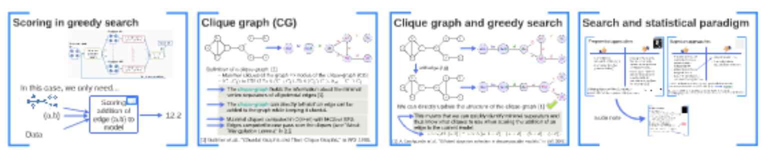
Graph theory



Evaluation - Scoring

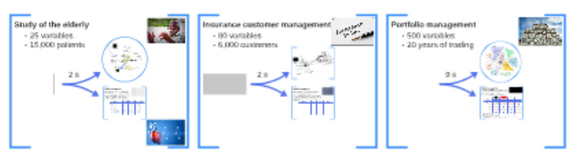
Break

Efficient search



The nitty-gritty

Use cases



Wrapping up!

Scalable learning of graphical models

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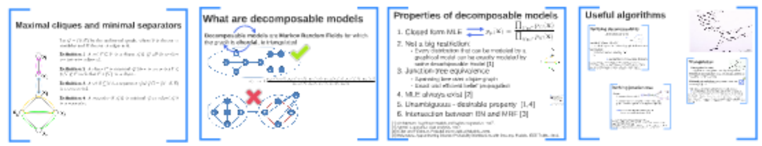
Graphical models 101

Graph theory



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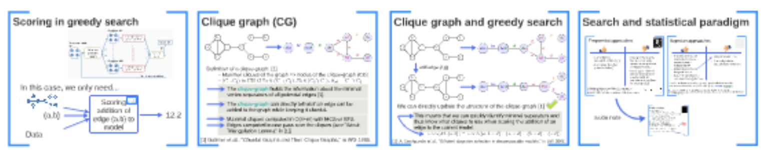

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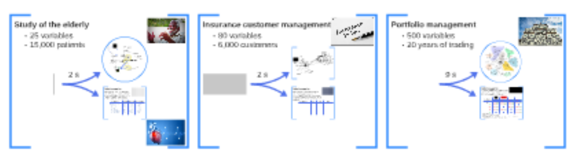
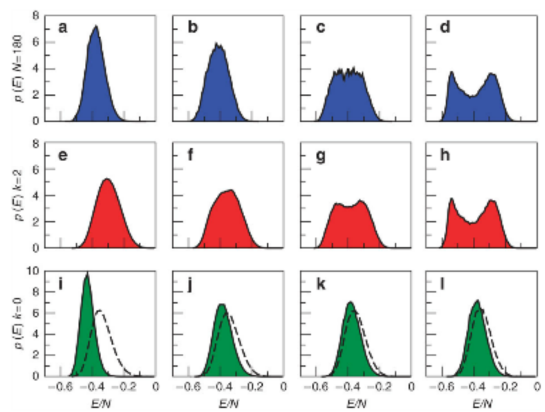


Wrapping up!

What is a probabilistic graphical model?

Probability theory

Quantifying uncertainty



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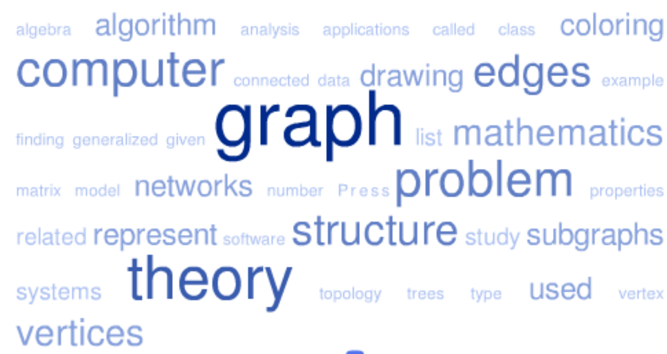
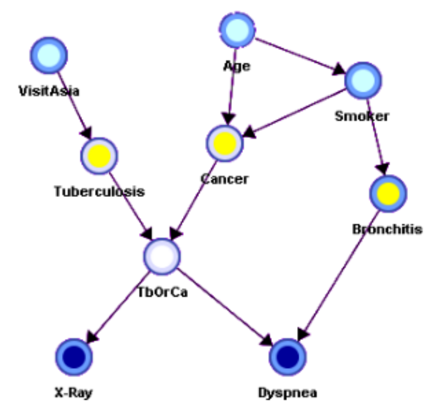


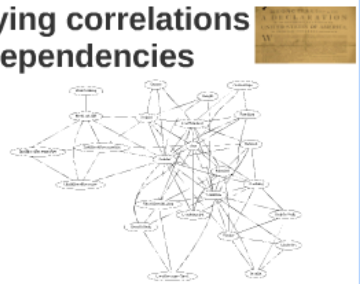
Not a black box +
Efficient algorithms



Aim: Compactly representing probability distributions

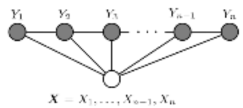
What are graphical models useful for?

Studying correlations & independencies



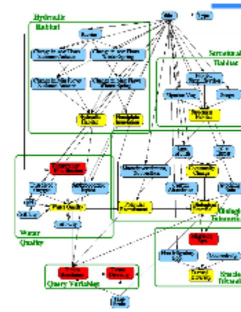
Simultaneously predicting multiple variables of...

Hidden Markov Models (HMM),
Maximum Entropy Markov Models (MEMM),
Conditional Random Fields (CRF),
Dense Random Fields (DRF),
...

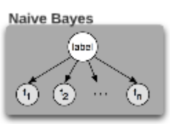


... the next sequence of words
... the class of sets of pixels
...

Causal Discovery & Inference

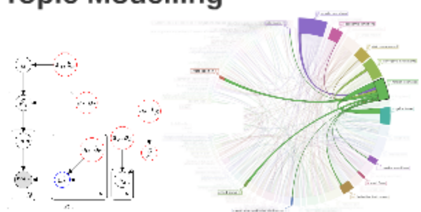


Classification



KDB, AODE, AnDE, ...

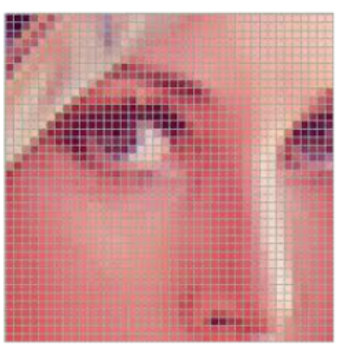
Topic Modelling

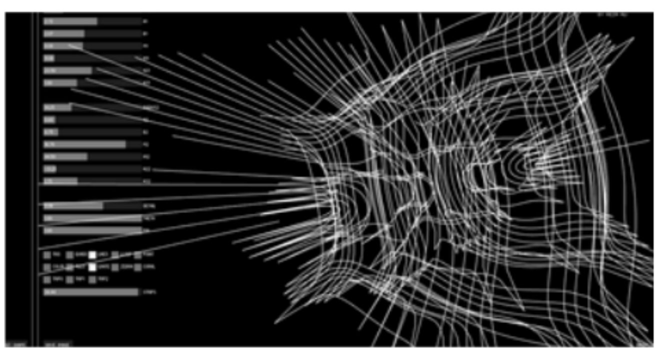


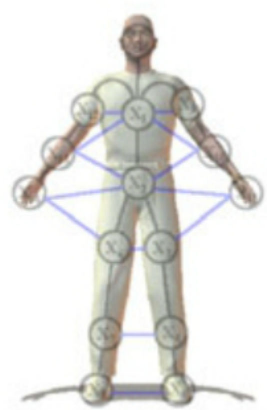
(c) Buntine and Mishra @KDD'14

...

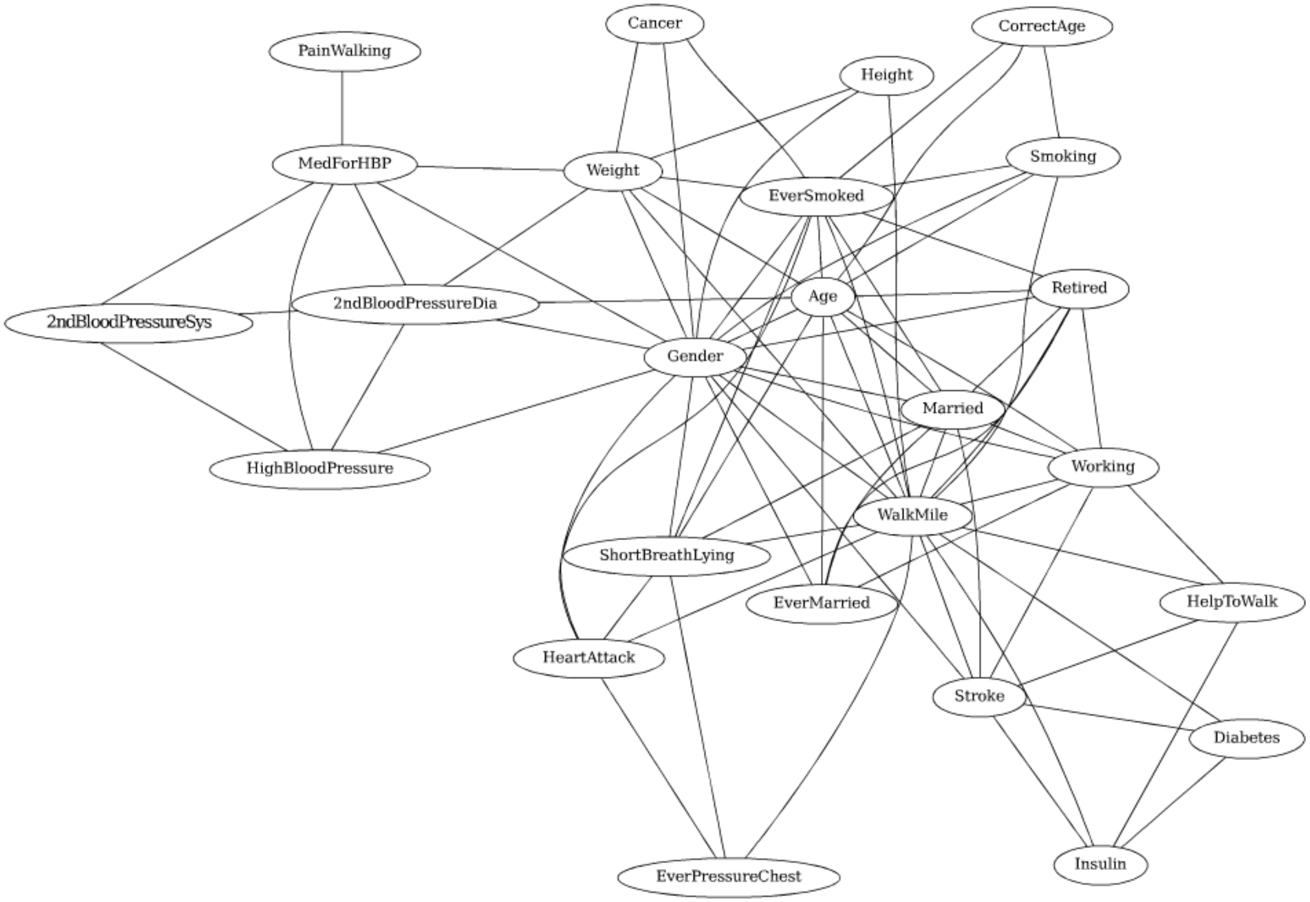
+ the thousands of applications of these methods...





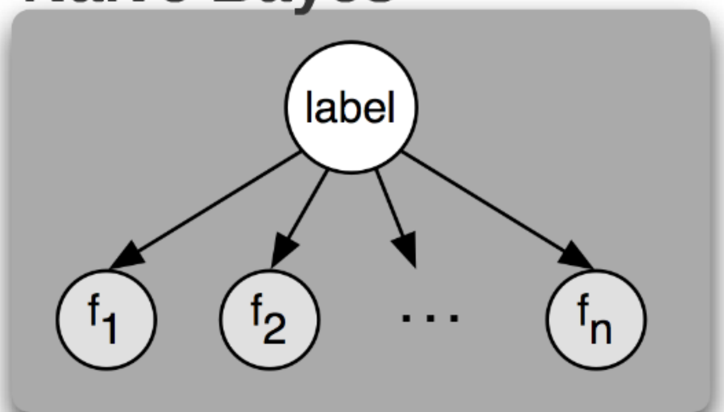


Studying correlations & independencies

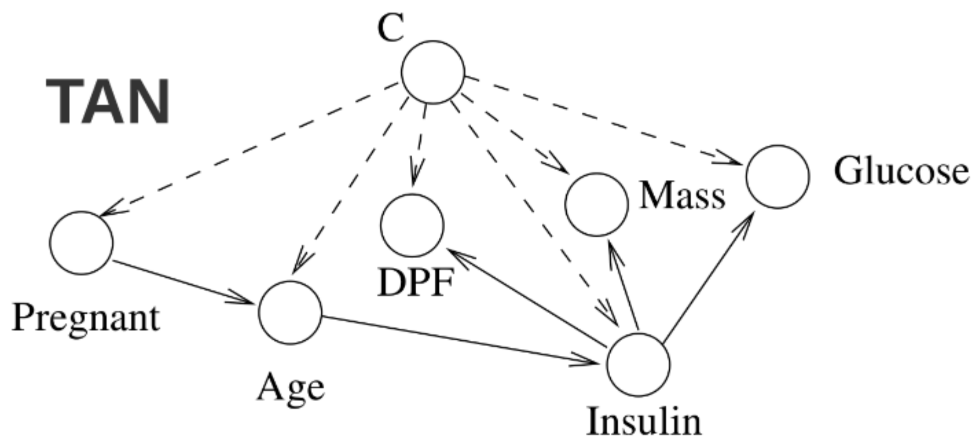


Classification

Naive Bayes



TAN



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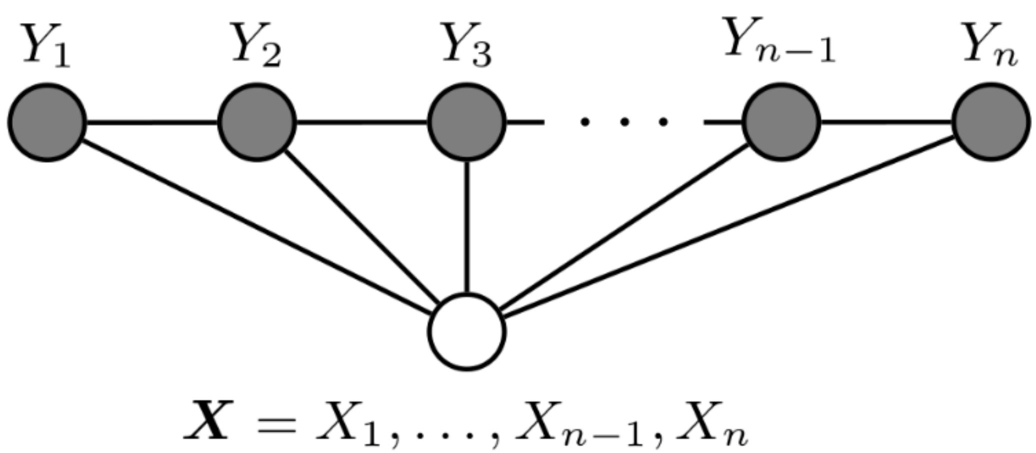

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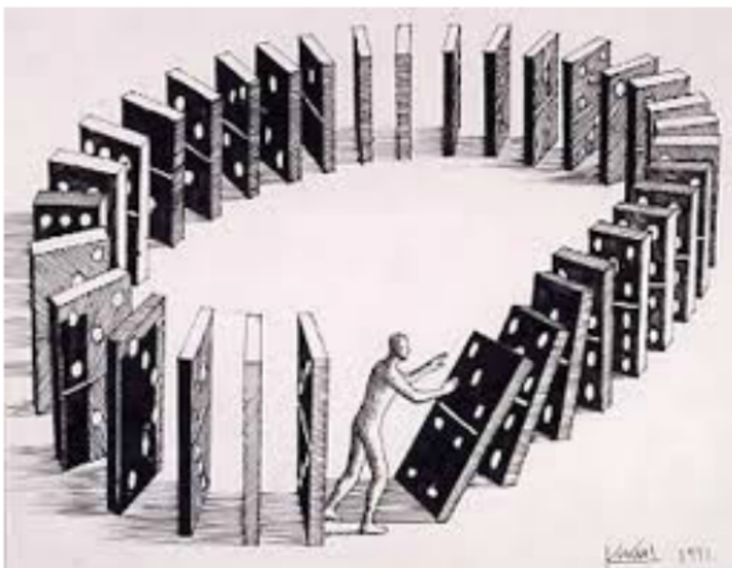
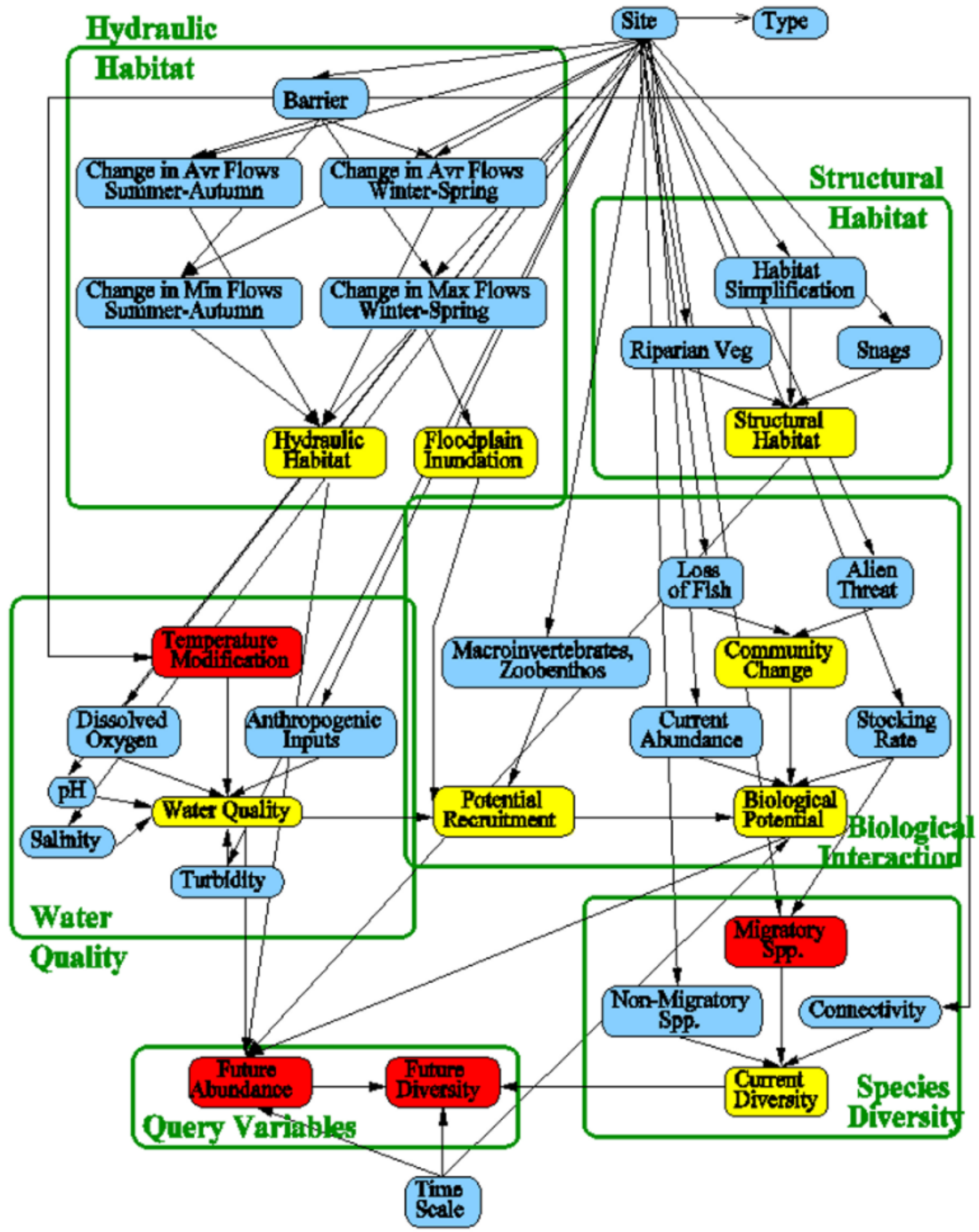
Causal Discovery & Inference





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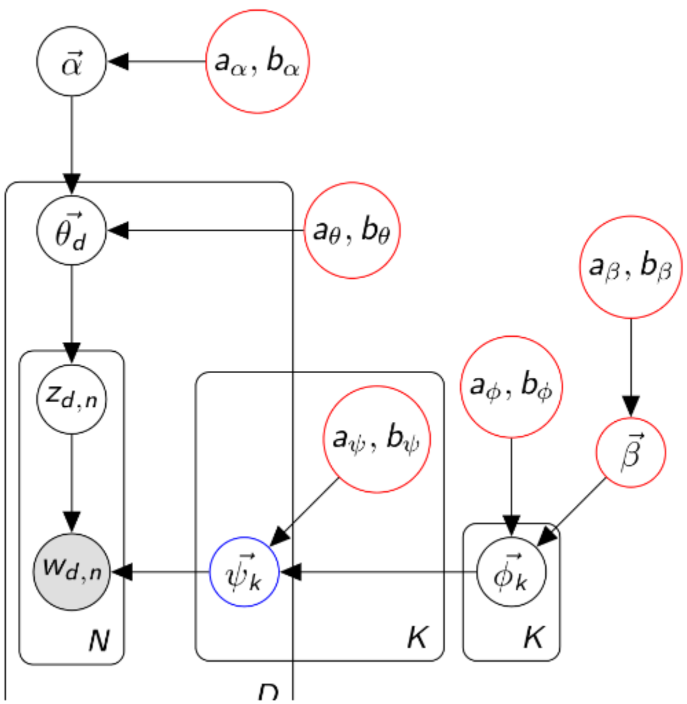

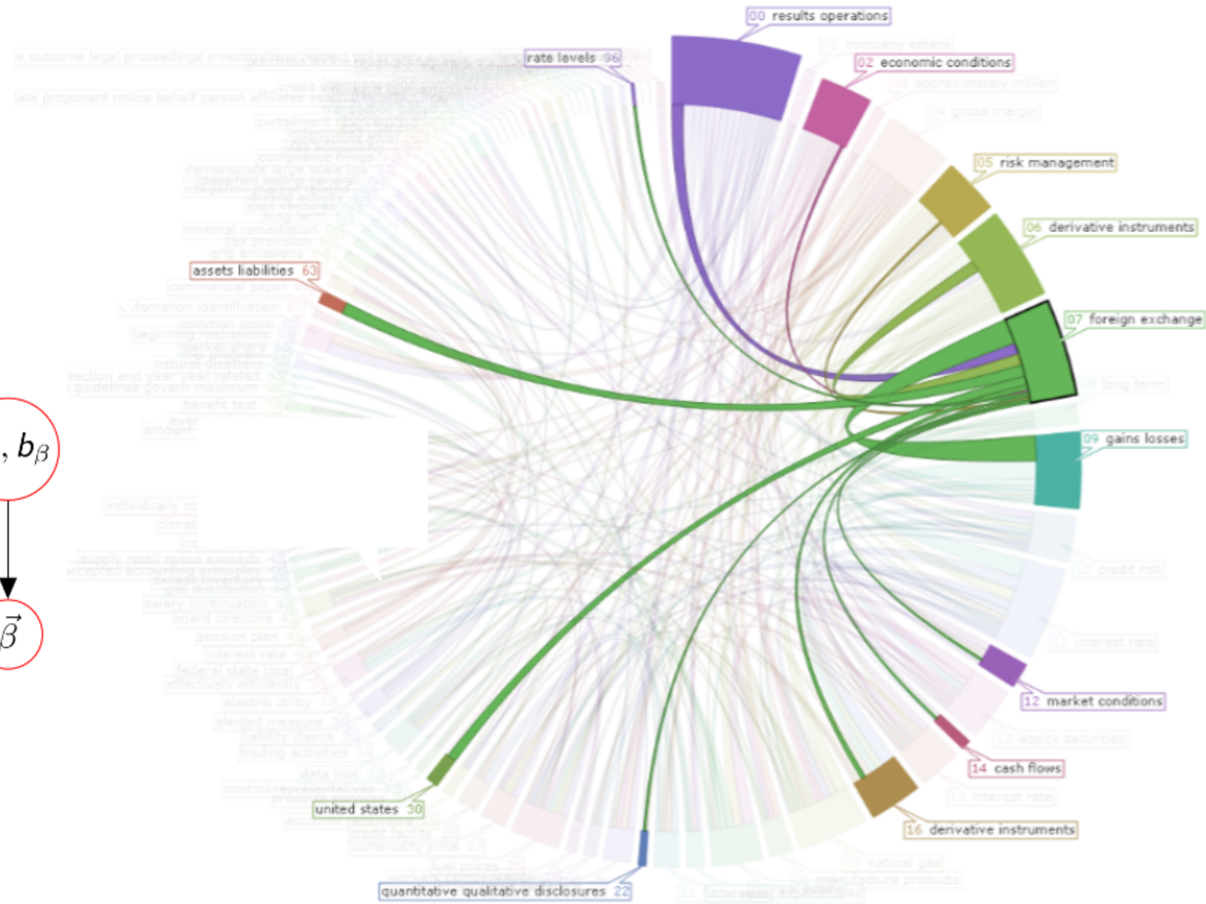

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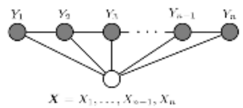

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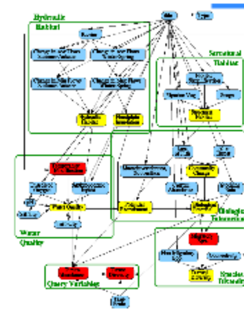

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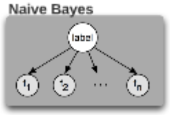

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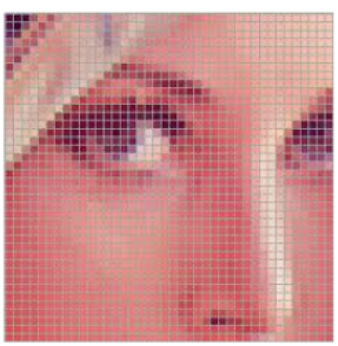
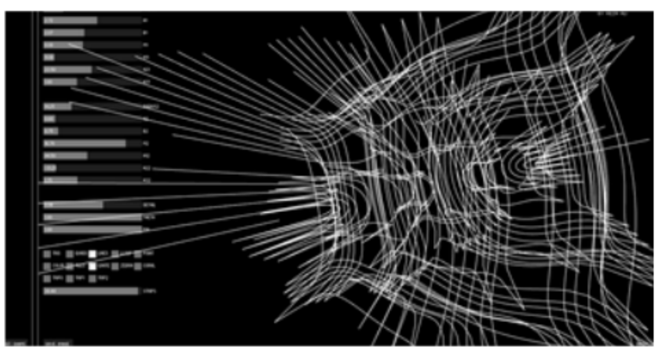
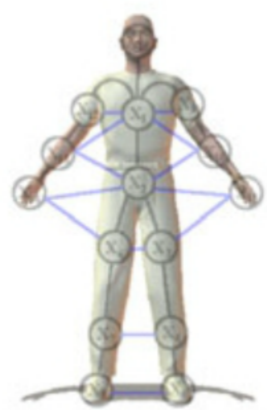




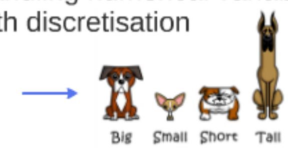


What we *will* and *will not* cover

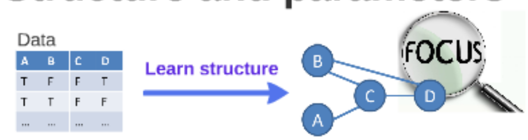
Discrete case

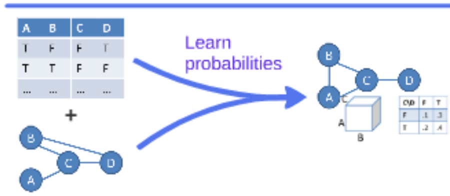
- Neat problem definitions

- Already very challenging
- Handling numerical variables with discretisation

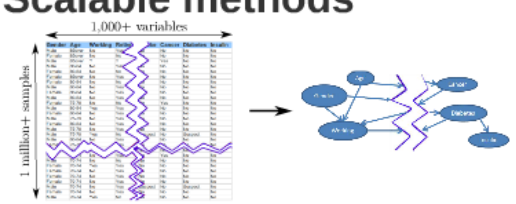


Structure and parameters





Scalable methods



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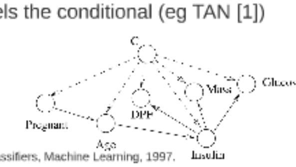
Modelling the joint distribution

Joint discrete distribution
→ $\mathrm{P}(X_1 = x_1, \ldots, X_n = x_n)$

Modelling conditional distribution
- ***Open problem*** *with intense research effort*
- Possible to use the joint to **approximate** a structure that models the conditional (eg TAN [1])



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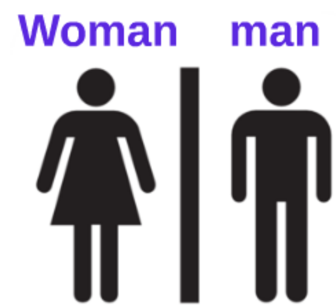

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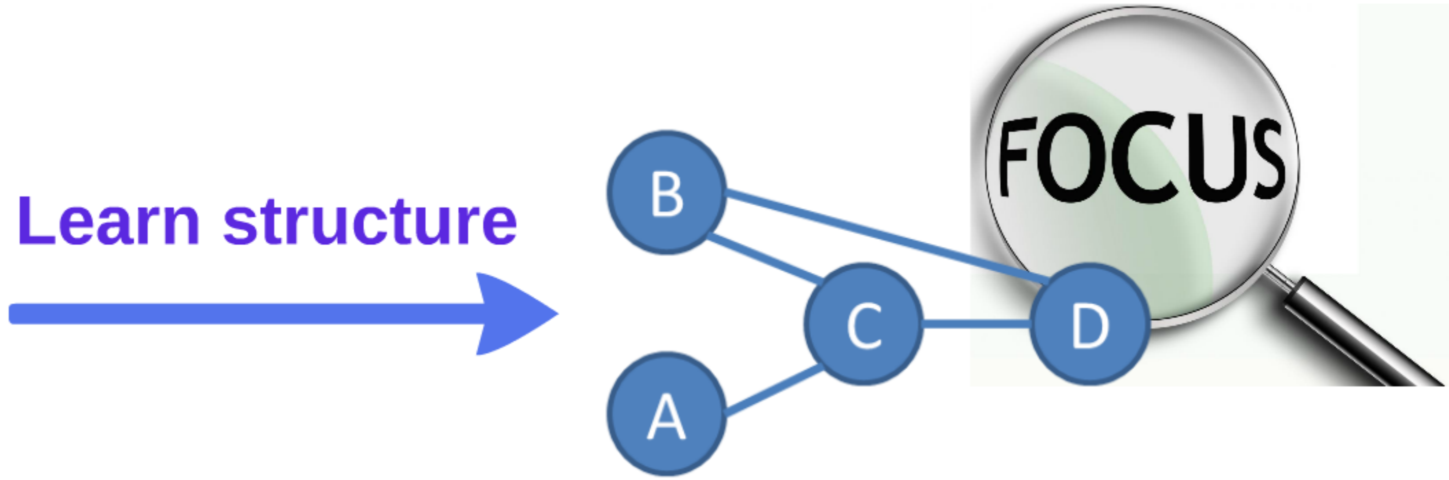
Structure and parameters

Data

A	B	C	D
T	F	F	T
T	T	F	F
…	…	…	…



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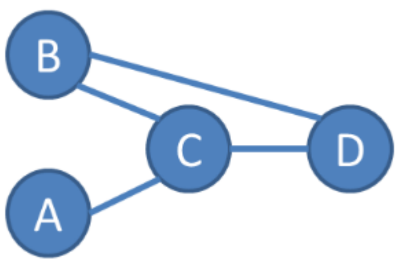
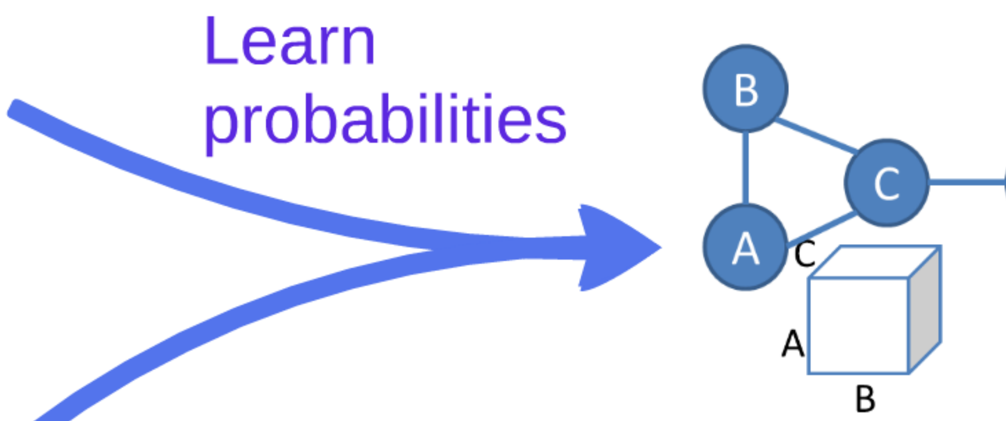
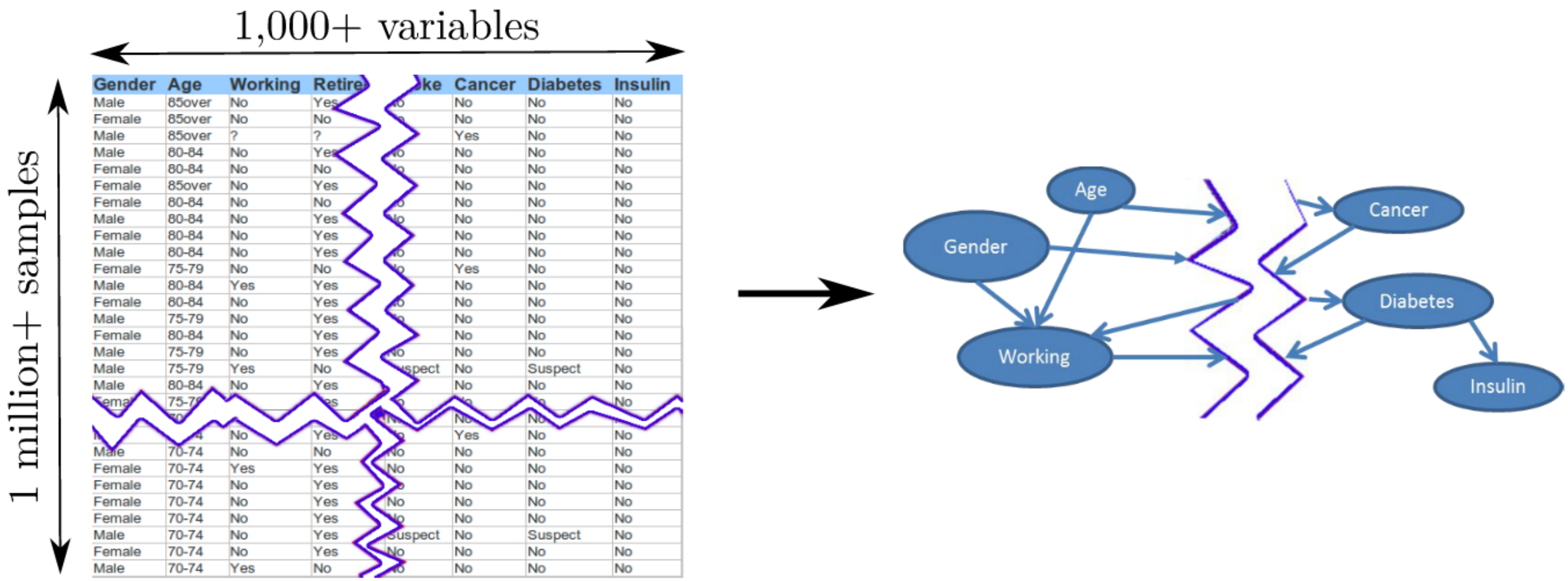
+





C\D	F	T
F	.1	.3
T	.2	.4

Scalable methods



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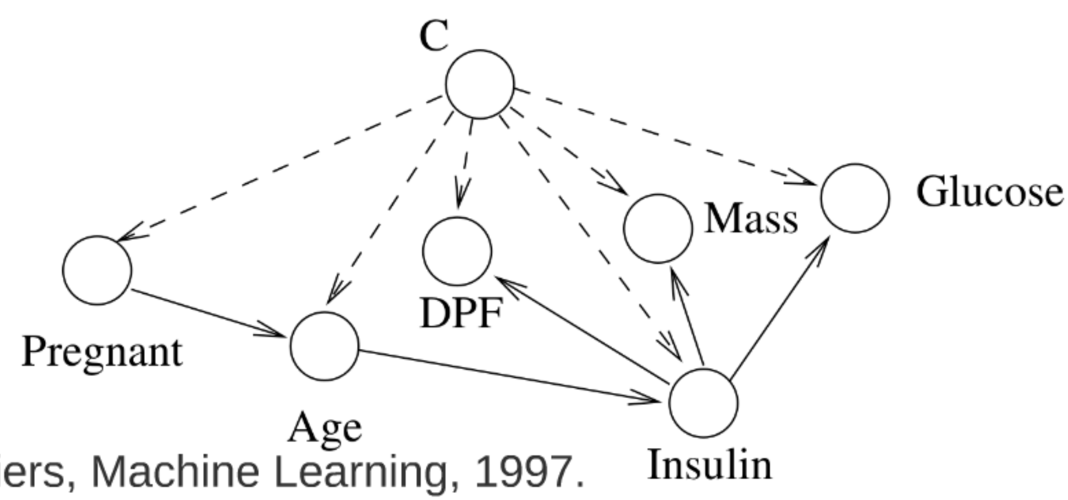

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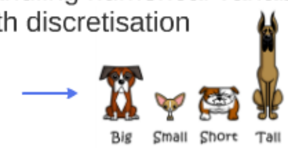

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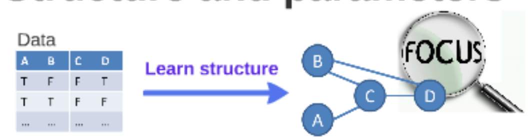

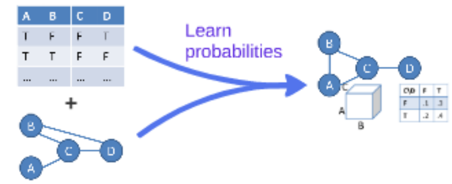

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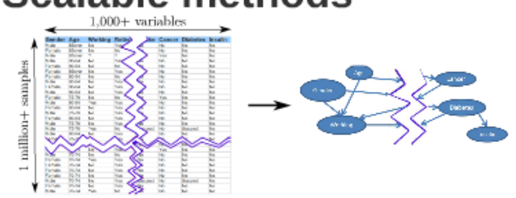

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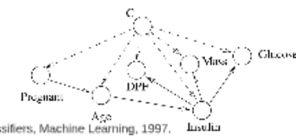

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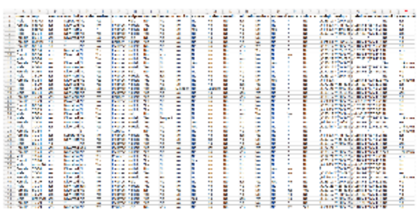
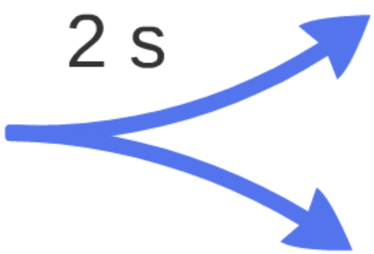
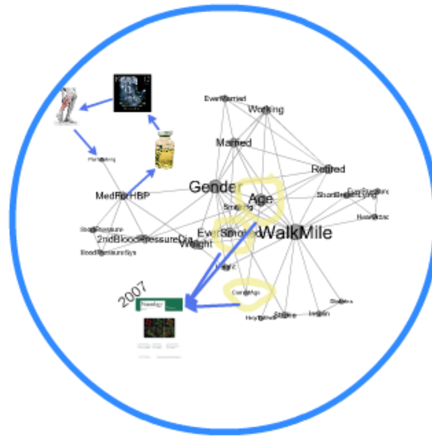
Study of the elderly

- 25 variables
- 15,000 patients



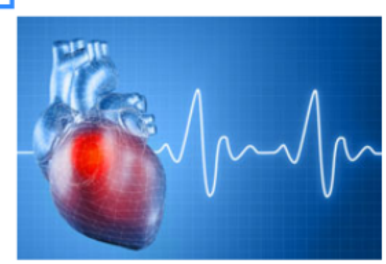




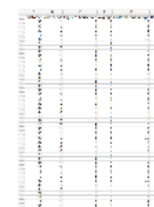
Belief propagation

New patient, Lan, is visiting her new GP; the GP wants to check her risk of getting a few diseases: stroke, diabetes, heart attack.

evidence	stroke	diabetes	heart attack
female under 70	5%	15%	10%
+ married	5%	15%	9%
+ smoking	7%	17%	12%
+ BP=17/10	8%	17%	13%
+ no help to walk	5%	16%	12%
+ quit smoking?	4%	14%	9%



Insu



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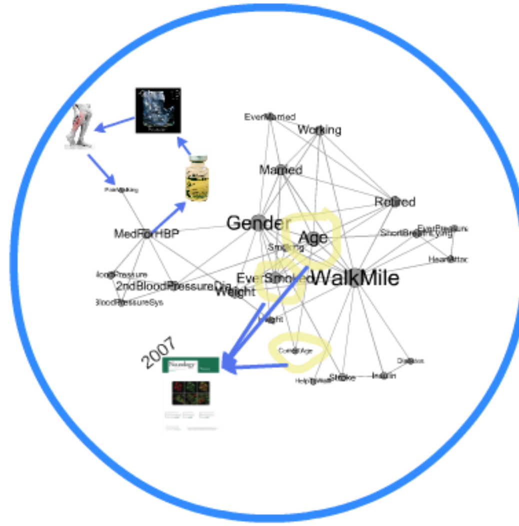

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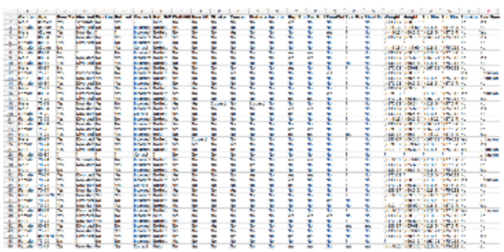

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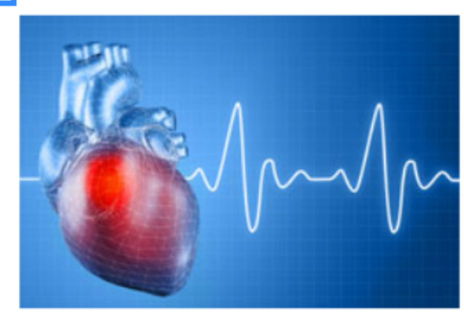

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y
1	**Gender**	**Age**	**EverMar**	**Married**	**Working**	**Retired**	**Correct**	**HelpToW**	**WalkMil**	**HeartAt**	**Stroke**	**Cancer**	**Diabete**	**Insulin**	**HighBlo**	**MedForl**	**PainWal**	**EverPre**	**ShortBr**	**Weight**	**Height**	**2ndBloo**	**2ndBloo**	**Smoking**	**EverSmo**
2	Male	85over	Yes	Separate	No	Yes	?	Help	No	No	No	No	No	No	Yes	No	No	?	No	\'(133-17	\'(60.5-65	?	?	No	Yes
3	Female	85over	Yes	Divorced	No	No	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	?	No	\'(-inf-13	\'(-inf-60.	\'(118.5-1	\'(37.5-75	No	No
4	Male	85over	Yes	NowMarr	?	?	Incorrect	NoHelp	No	No	No	Yes	No	No	No	No	Yes	?	No	\'(-inf-13	\'(60.5-65	\'(118.5-1	\'(37.5-75	No	Yes
5	Male	80-84	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	?	No	\'(133-17	\'(69.5-in	\'(167-21	\'(75-112	No	Yes
6	Female	80-84	Yes	Divorced	No	No	Incorrect	Help	No	No	No	No	No	No	No	No	No	?	No	?	?	?	?	No	No
7	Female	85over	No	?	No	Yes	Correct	NoHelp	Yes	No	No	No	No	No	No	No	No	?	No	\'(-inf-13	\'(-inf-60.	\'(118.5-1	\'(75-112	No	No
8	Female	80-84	No	?	No	No	Incorrect	NoHelp	No	No	No	No	No	No	Yes	Yes	No	?	No	\'(133-17	\'(60.5-65	\'(118.5-1	\'(37.5-75	No	No
9	Male	80-84	Yes	NowMarr	No	Yes	Incorrect	NoHelp	No	No	No	No	No	No	Yes	Yes	No	?	No	\'(133-17	\'(65-69.5	\'(167-21	\'(75-112	No	Yes
10	Female	80-84	Yes	Divorced	No	Yes	Incorrect	NoHelp	No	Yes	No	No	No	No	No	No	No	No	No	\'(133-17	\'(60.5-65	\'(118.5-1	\'(75-112	No	No
11	Male	80-84	No	?	No	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	Yes	No	No	?	No	\'(172-21	\'(65-69.5	\'(167-21	\'(75-112	No	No
12	Female	75-79	Yes	Divorced	No	No	Incorrect	NoHelp	No	No	No	Yes	No	No	No	No	Yes	?	No	\'(133-17	\'(60.5-65	?	?	Yes	Unknown
13	Male	80-84	Yes	NowMarr	Yes	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	?	No	\'(133-17	\'(60.5-65	\'(118.5-1	\'(75-112	Yes	Unknown
14	Female	80-84	Yes	Divorced	No	Yes	Incorrect	Help	No	Yes	No	No	No	No	No	No	No	?	No	\'(172-21	\'(60.5-65	\'(118.5-1	\'(37.5-75	No	No
15	Male	75-79	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	Yes	No	No	No	No	No	No	No	?	No	\'(133-17	\'(69.5-in	?	?	No	No
16	Female	80-84	Yes	Divorced	No	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	Yes	?	No	\'(-inf-13	?	\'(118.5-1	\'(75-112	Yes	Unknown
17	Male	75-79	Yes	Divorced	No	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	Yes	Yes	No	?	No	\'(172-21	\'(65-69.5	\'(167-21	\'(75-112	No	Yes
18	Male	75-79	Yes	NowMarr	Yes	No	Incorrect	NoHelp	Yes	Yes	Suspect	No	Suspect	No	No	No	No	?	No	\'(172-21	\'(69.5-in	\'(118.5-1	\'(37.5-75	No	No
19	Male	80-84	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	Yes	Yes	No	?	No	\'(133-17	\'(65-69.5	\'(118.5-1	\'(75-112	No	Yes
20	Female	75-79	Yes	NowMarr	No	Yes	Correct	NoHelp	Yes	No	No	No	No	No	Yes	No	No	?	No	\'(133-17	\'(60.5-65	\'(118.5-1	\'(75-112	No	No
21	Female	75-79	Yes	Divorced	No	No	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	?	No	\'(-inf-13	\'(60.5-65	\'(-inf-118	\'(37.5-75	No	No
22	Female	80-84	Yes	NowMarr	No	No	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	?	No	\'(133-17	\'(60.5-65	\'(167-21	\'(37.5-75	No	No
23	Female	75-79	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	Yes	Yes	No	?	No	\'(-inf-13	\'(60.5-65	\'(167-21	\'(37.5-75	No	No
24	Male	75-79	Yes	Divorced	No	Yes	Incorrect	Help	No	No	No	No	No	No	Yes	Yes	No	Yes	No	\'(133-17	\'(69.5-in	\'(-inf-118	\'(37.5-75	No	Yes
25	Male	80-84	Yes	Divorced	Yes	Yes	Incorrect	NoHelp	Yes	Suspect	No	No	No	No	No	No	No	?	No	\'(133-17	\'(60.5-65	\'(167-21	\'(75-112	Yes	Unknown
26	Female	70-74	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	Yes	No	Yes	No	No	No	No	No	?	No	\'(-inf-13	\'(-inf-60.	\'(118.5-1	\'(37.5-75	No	Yes
27	Male	70-74	Yes	Divorced	No	Yes	Incorrect	NoHelp	Yes	No	No	Yes	No	No	No	No	No	?	No	\'(211-inf	\'(65-69.5	\'(118.5-1	\'(75-112	Yes	Unknown
28	Female	80-84	?	?	?	?	Correct	?	No	No	No	No	No	No	No	No	No	?	No	?	?	?	?	?	Unknown
29	Female	70-74	Yes	Separate	No	No	Incorrect	NoHelp	Yes	No	No	No	No	No	Yes	Yes	No	?	No	\'(-inf-13	\'(-inf-60.	\'(118.5-1	\'(37.5-75	No	Yes
30	Male	70-74	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	No	No	No	Yes	No	No	No	No	?	No	\'(133-17	\'(65-69.5	\'(118.5-1	\'(37.5-75	No	No
31	Male	80-84	No	?	No	Yes	Incorrect	NoHelp	Yes	No	Yes	No	No	No	No	No	No	?	No	\'(133-17	\'(60.5-65	\'(118.5-1	\'(75-112	No	Yes
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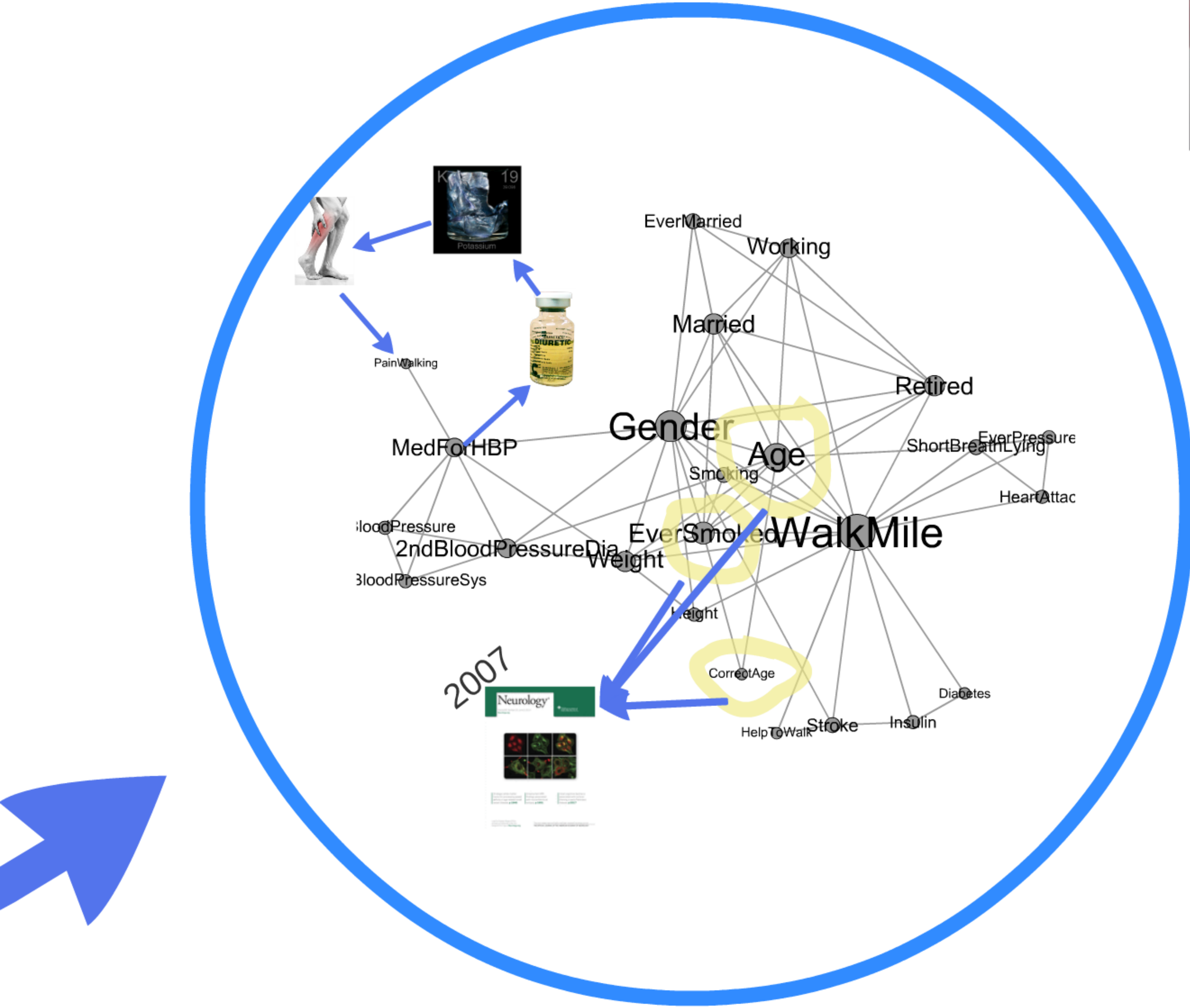
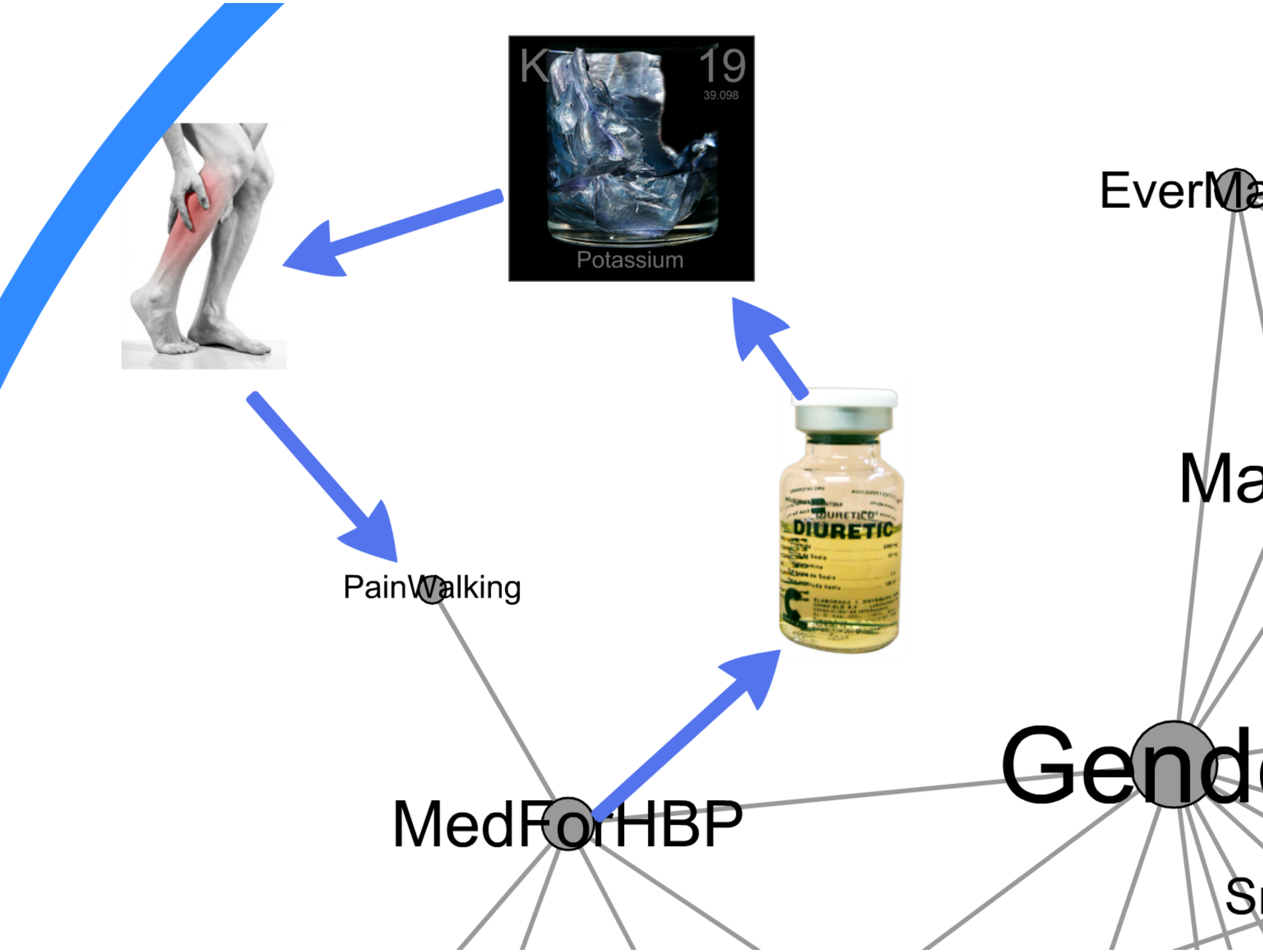
33	Female	70-74	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	Yes	No	No	?	No	\'(133-17	\'(60.5-65	\'(118.5-1	\'(75-112	Yes	Unknown
34	Male	70-74	Yes	NowMarr	No	Yes	Correct	NoHelp	Yes	No	No	No	No	No	Yes	No	No	?	No	\'(172-21	\'(65-69.5	\'(118.5-1	\'(37.5-75	No	Yes
35	Female	70-74	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	No	No	Yes	No	No	No	No	No	?	No	\'(133-17	\'(60.5-65	\'(118.5-1	\'(75-112	No	No
36	Male	under70	Yes	NowMarr	Yes	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	?	No	\'(-inf-13	\'(69.5-in	\'(-inf-118	\'(37.5-75	No	No
37	Female	70-74	Yes	Divorced	No	No	Incorrect	NoHelp	No	No	No	No	No	No	No	No	Yes	No	No	?	\'(60.5-65	\'(118.5-1	\'(37.5-75	No	No
38	Male	70-74	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	No	No	\'(211-inf	\'(65-69.5	\'(118.5-1	\'(37.5-75	No	Yes
39	Female	80-84	Yes	Divorced	No	Yes	Incorrect	NoHelp	Yes	Yes	No	No	No	No	No	No	No	?	No	\'(-inf-13	\'(60.5-65	\'(118.5-1	\'(37.5-75	Yes	Unknown
40	Female	75-79	Yes	Divorced	No	No	Incorrect	Help	Yes	No	No	No	No	No	Yes	Yes	Yes	No	No	\'(133-17	\'(60.5-65	\'(118.5-1	\'(75-112	No	No
41	Male	70-74	Yes	NowMarr	Yes	No	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	?	No	\'(172-21	\'(65-69.5	\'(118.5-1	\'(75-112	No	No
42	Female	80-84	Yes	Divorced	No	No	Incorrect	NoHelp	No	No	Yes	No	No	No	No	No	No	Yes	Yes	\'(133-17	\'(65-69.5	\'(118.5-1	\'(37.5-75	No	No
43	Female	75-79	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	No	No	\'(133-17	\'(-inf-60.	\'(167-21	\'(75-112	No	No
44	Male	under70	Yes	Divorced	No	Yes	Incorrect	NoHelp	Yes	No	No	Yes	No	No	Yes	Yes	No	No	No	\'(133-17	\'(60.5-65	\'(118.5-1	\'(75-112	No	Yes
45	Female	75-79	No	?	No	No	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	?	No	?	\'(60.5-65	\'(167-21	\'(75-112	No	No
46	Female	75-79	Yes	NowMarr	No	Yes	Correct	Help	No	Yes	No	No	No	No	Yes	Yes	No	Yes	Yes	\'(-inf-13	\'(60.5-65	\'(118.5-1	\'(75-112	No	No


K
19
Potassium
DIURETIC
PainWalking
MedForHBP
BloodPressure
2ndBloodPressureDia
BloodPressureSys
EverMarried
Working
Married
Retired
Gender
Age
Smoking
EverSmoked
WalkMile
Weight
Height
ShortBreathLying
EverPressure
HeartAttac
CorrectAge
Diabetes
Insulin
Stroke
HelpToWalk
2007
Neurology


K
19
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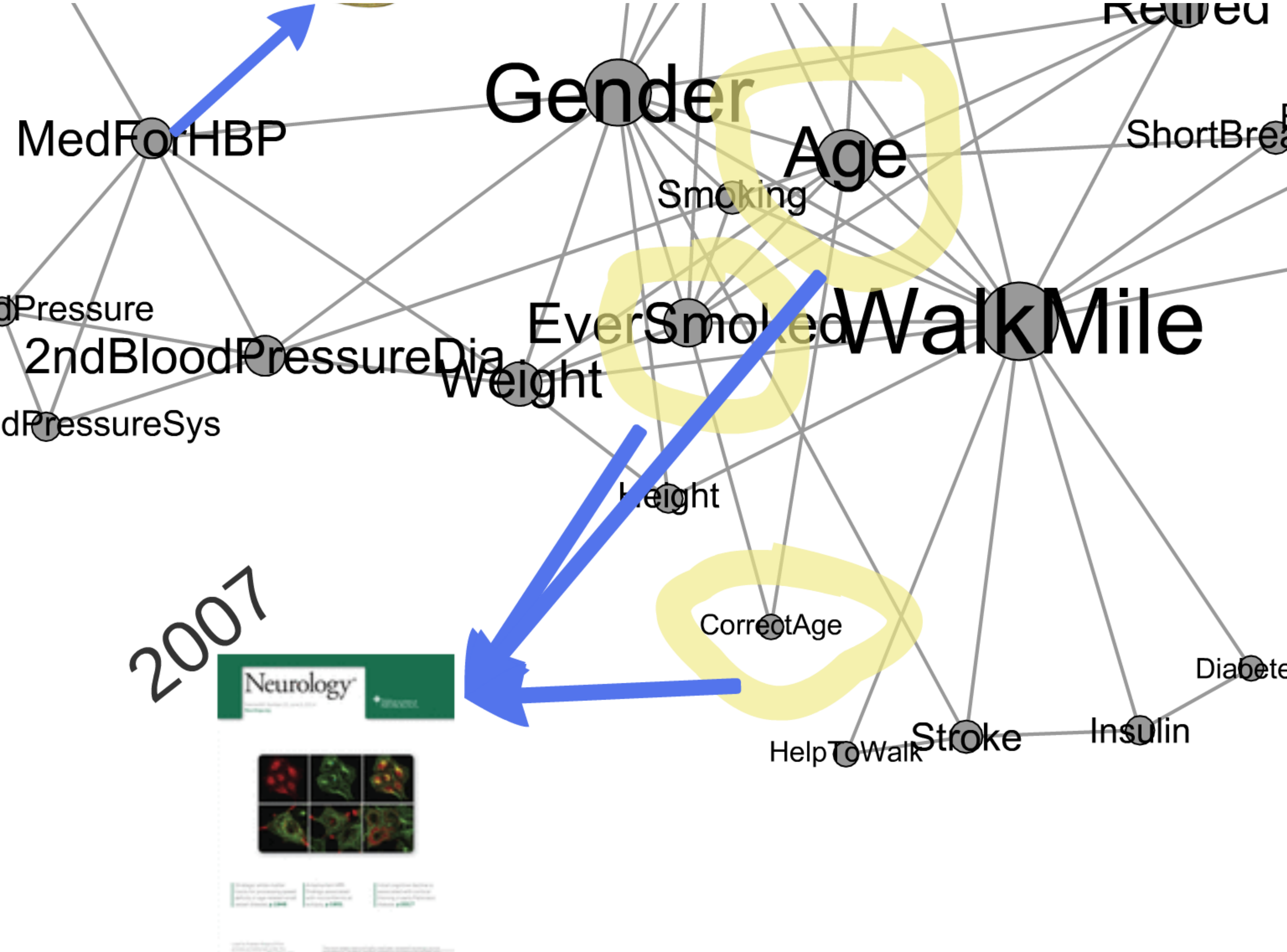
Potassium
DIURETIC
PainWalking
MedForHBP


Retired
Gender
MedForHBP
Age
ShortBrea
Smoking
dPressure
EverSmoked
WalkMile
2ndBloodPressureDia
Weight
dPressureSys
Height
2007
Neurology
CorrectAge
Diabete
Stroke
Insulin
HelpToWalk

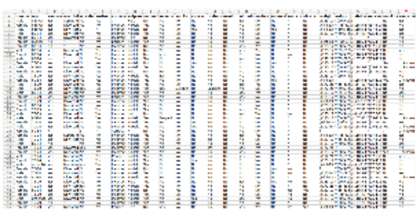
Belief propagation

New patient, Lan, *is visiting her new GP; the GP wants to check her risk of getting a few diseases: stroke, diabetes, heart attack.*

evidence	stroke	diabetes	heart attack
female under 70	*5%*	*15%*	*10%*
+ married	*5%*	*15%*	*9%*
+ smoking	*7%*	*17%*	*12%*
+ BP=17/10	*8%*	*17%*	*13%*
+ no help to walk	*5%*	*16%*	*12%*
+ quit smoking?	*4%*	*14%*	*9%*

Study of the elderly

- 25 variables
- 15,000 patients



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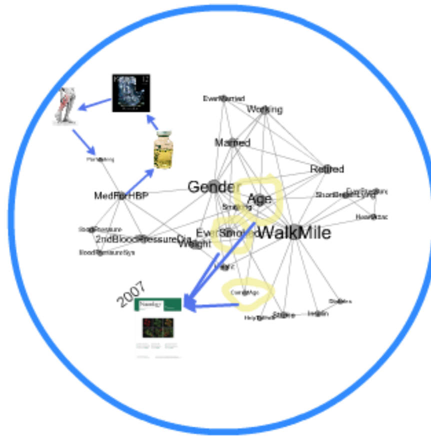

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+ no help to walk	*5%*	*16%*	*12%*
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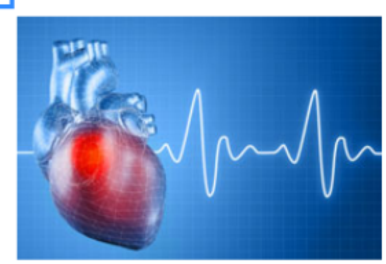
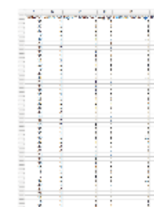


Insu

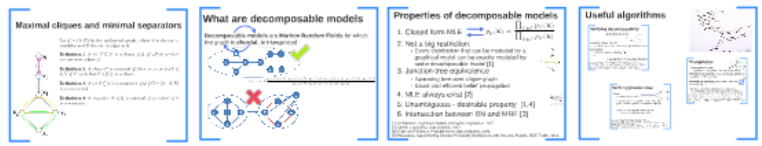


Scalable learning of graphical models

Introduction - Motivation

Graphical models 101

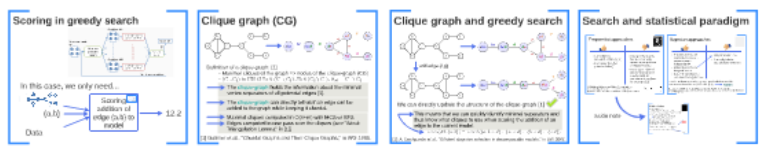
Graph theory



Evaluation - Scoring

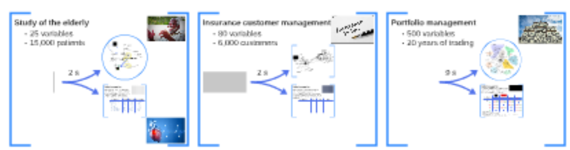
Break

Efficient search



The nitty-gritty

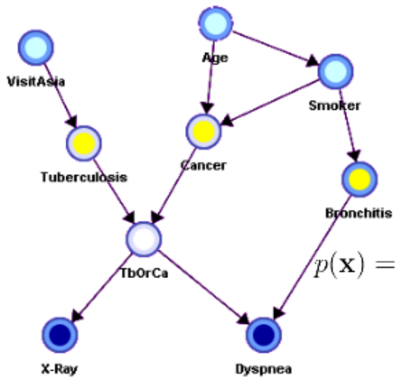
Use cases



Wrapping up!

Classes of graphical models

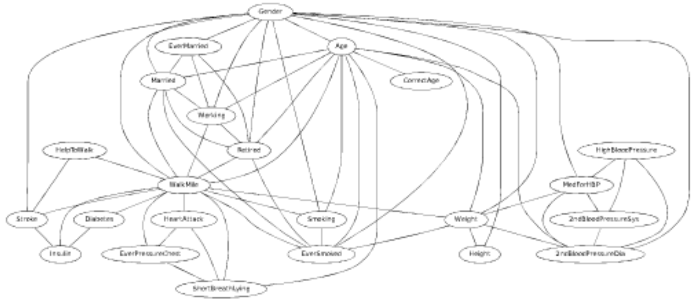
Bayesian Network



$$p(\mathbf{x}) = \prod_{v \in V} p\left(\mathbf{x}_v | \mathbf{x}_{parents(v)}\right)$$

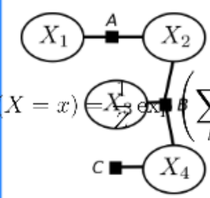
Possible causal interpretation

Markov networks or Markov Random Fields



Special case of log-linear models that have the property of being graphical

Factor graphs

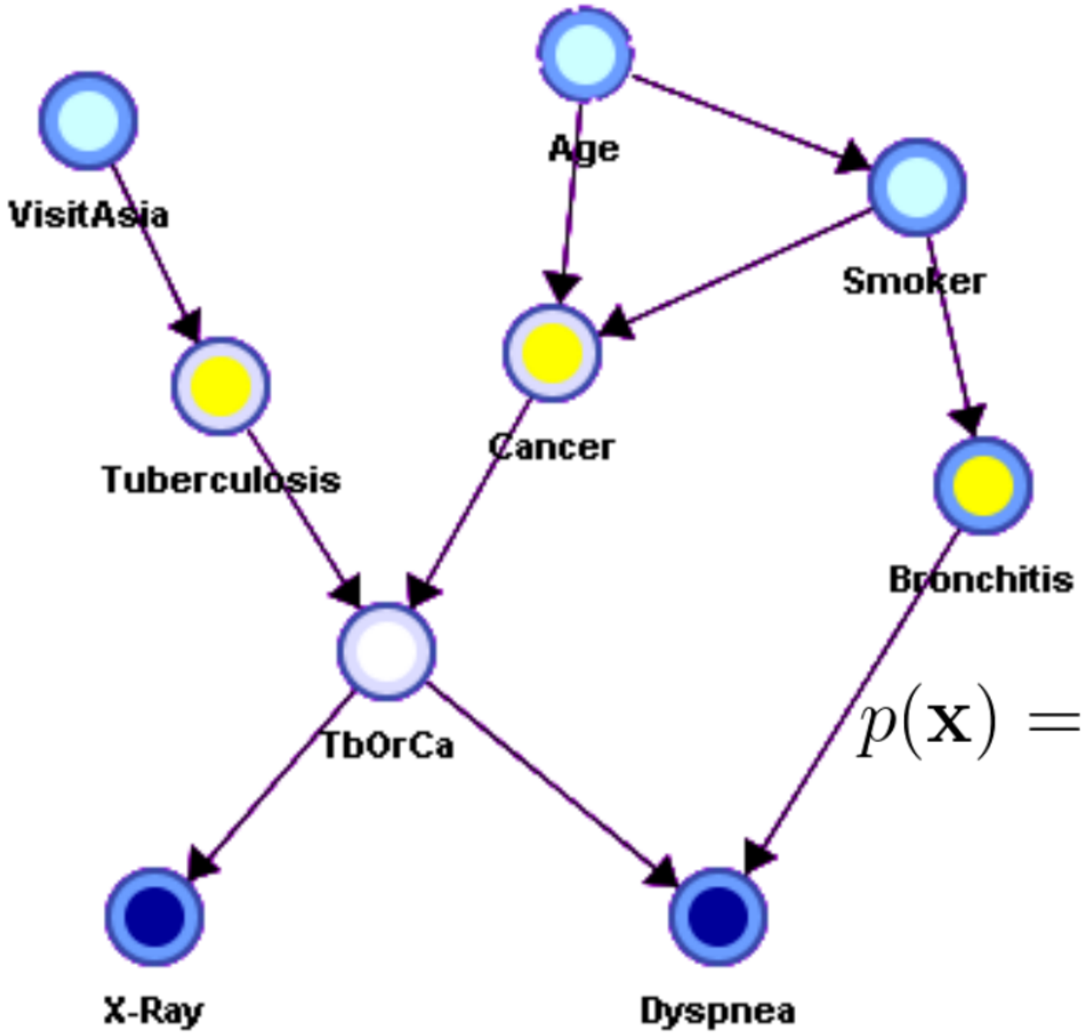


$$p(\mathbf{x}) = \prod_{j=1}^{m} f_j(S_j)$$

$$P(X = x) = \frac{1}{Z} \exp\left(\sum_k w_k^\top f_k(x_{\{k\}})\right)$$

...

Bayesian Network



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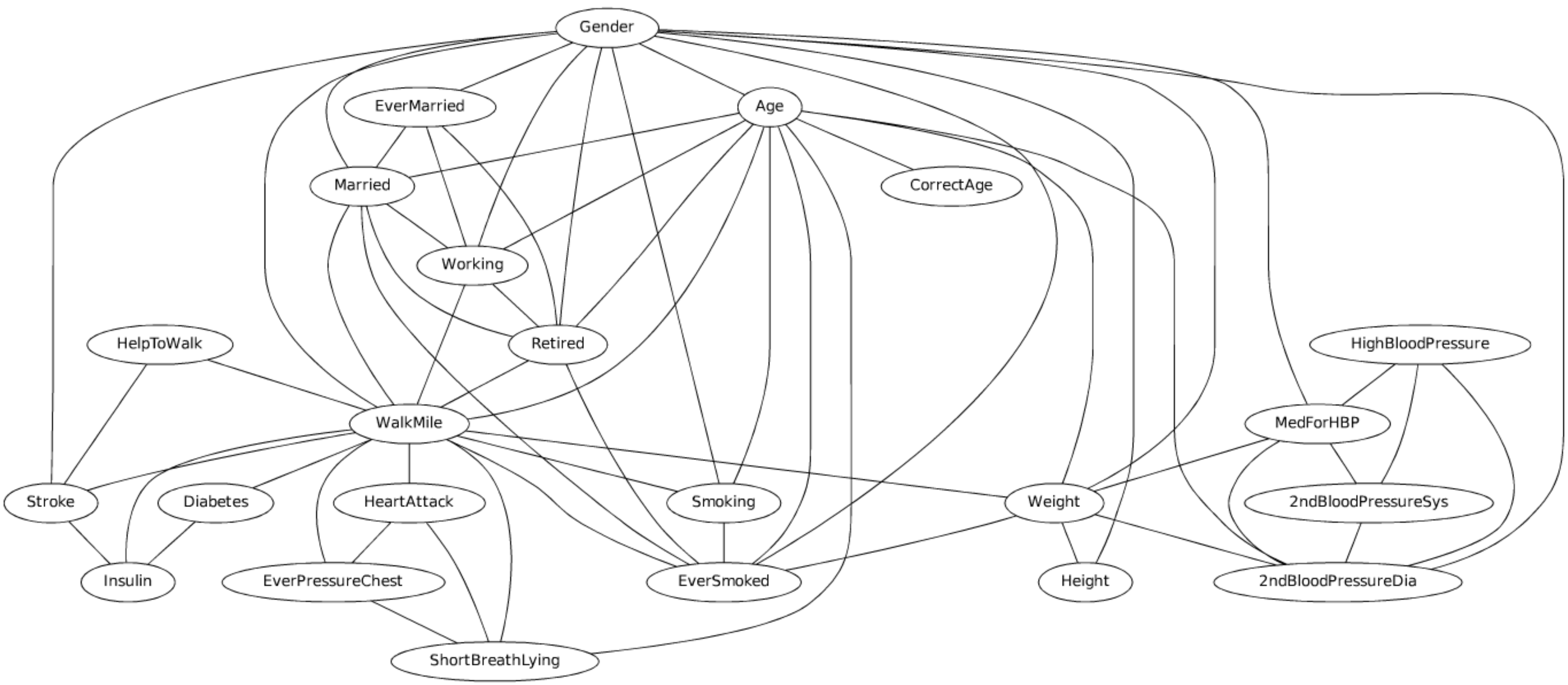


Special case of log-linear models that have the property of being graphical

A simple example of structure learning

Hill-climbing search on MRF using AIC

Current AIC=1258

edge addition	AIC
sex-died	825
class-sex	851
class-died	1,083
class-age	1,115
sex-age	1,236
age-died	1,240

73% of women survived vs 1% of men

Current AIC=825

edge addition	AIC
class-sex	419
class-died	650
class-age	683
sex-age	804
age-died	808

97% of the crew were men

Current AIC=419

edge addition	AIC
class-age	276
class-died	319
sex-age	397
age-died	401

2% of 1st class were children
8% of 2nd class were children
11% of 3rd class were children
0% of the crew were children

Current AIC=276

edge addition	AIC
class-died	117
age-died	267
sex-age	272

62% of the people in 1st class survived
41% of the people in 2nd class survived
25% of the people in 3rd class survived
24% of the crew survived

Current AIC=117

edge addition	AIC
age-died	70
sex-age	95

52% of children survived
31% of adults survived

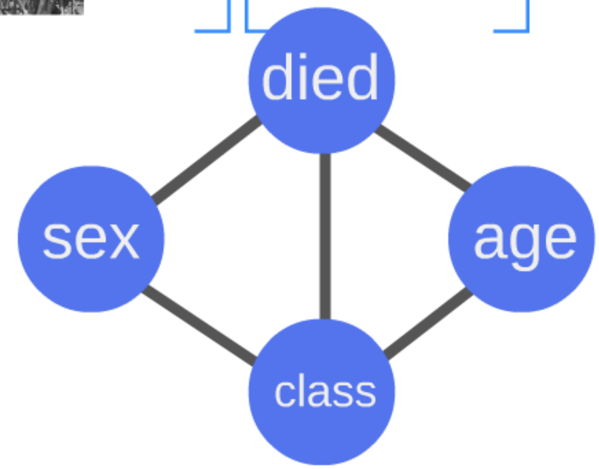
Current AIC=70

edge addition	AIC
sex-age	72

=> stop

To redo this experiment

Just 4 lines of R

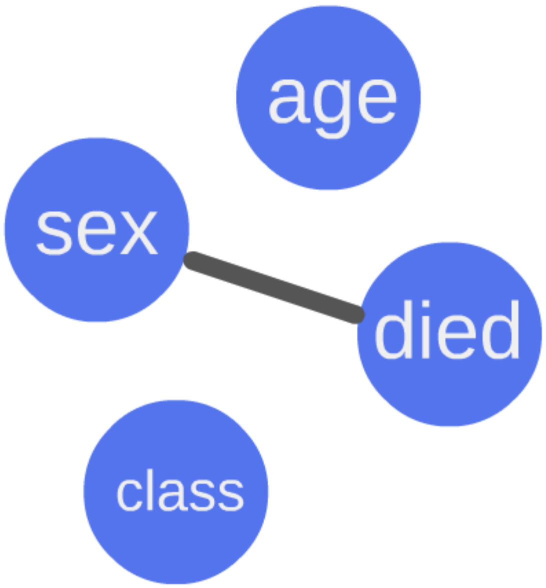


To predict survival:

- Yes, age matters
- Yes, class matters
- Yes, sex matters
- Yes, class and sex together matter (eg knowing that a particular man was in 1st class or crew)
- Yes, class and age together matter (eg knowing that a particular child was in 1st or 3rd class)
- No, sex and age don't matter together for a particular class (within each class, age and sex interact with survival independently of one another)

	A	B	C	D	E
1	Class	Sex	Age	Survived	Frequency
2	1st	Male	Child	No	0
3	2nd	Male	Child	No	0
4	3rd	Male	Child	No	35
5	Crew	Male	Child	No	0
6	1st	Female	Child	No	0
7	2nd	Female	Child	No	0
8	3rd	Female	Child	No	17
9	Crew	Female	Child	No	0
10	1st	Male	Adult	No	118
11	2nd	Male	Adult	No	154
12	3rd	Male	Adult	No	387
13	Crew	Male	Adult	No	670
14	1st	Female	Adult	No	4
15	2nd	Female	Adult	No	13
16	3rd	Female	Adult	No	89
17	Crew	Female	Adult	No	3
18	1st	Male	Child	Yes	5
19	2nd	Male	Child	Yes	11
20	3rd	Male	Child	Yes	13
21	Crew	Male	Child	Yes	0
22	1st	Female	Child	Yes	1
23	2nd	Female	Child	Yes	13
24	3rd	Female	Child	Yes	14
25	Crew	Female	Child	Yes	0
26	1st	Male	Adult	Yes	57
27	2nd	Male	Adult	Yes	14
28	3rd	Male	Adult	Yes	75
29	Crew	Male	Adult	Yes	192
30	1st	Female	Adult	Yes	140
31	2nd	Female	Adult	Yes	80
32	3rd	Female	Adult	Yes	76
33	Crew	Female	Adult	Yes	20

Current AIC=1258



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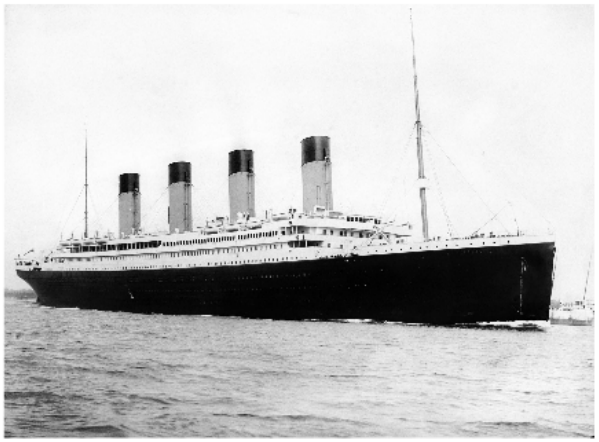

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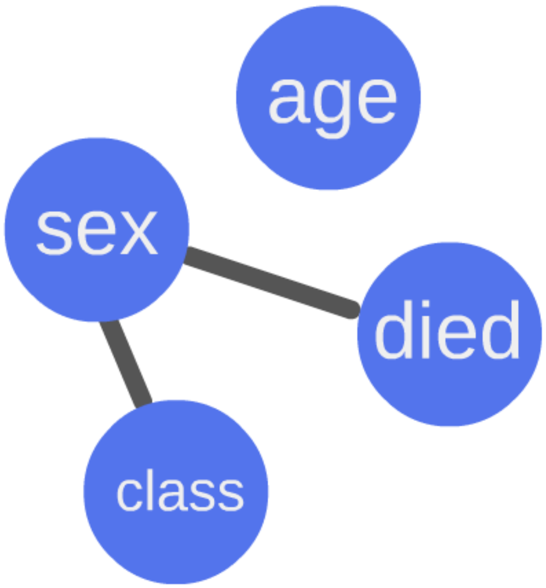

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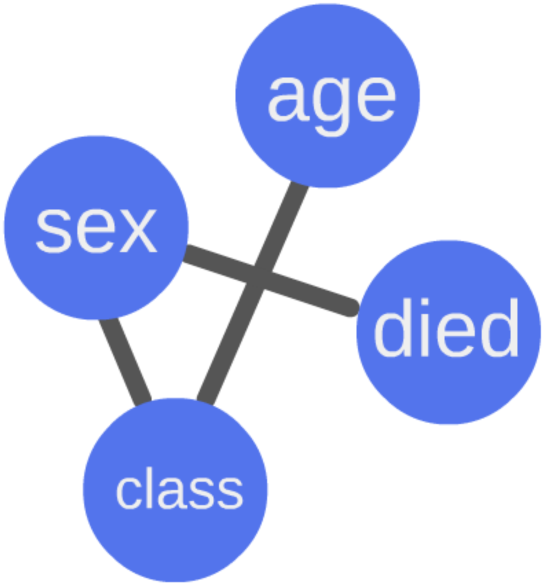

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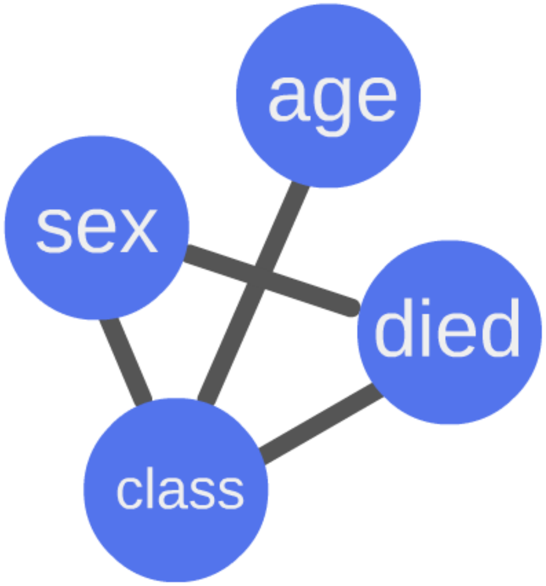

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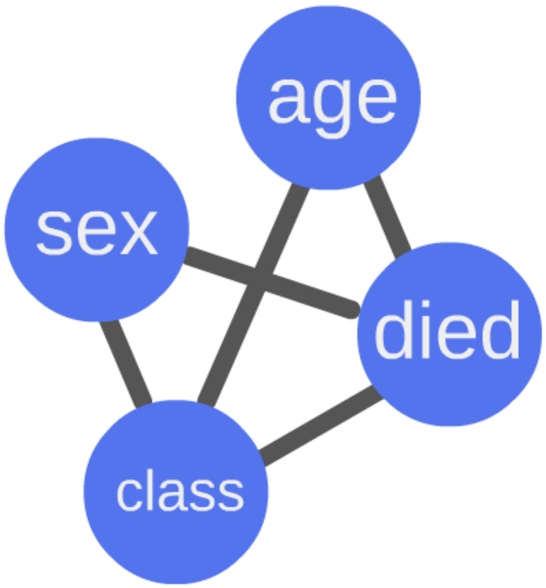

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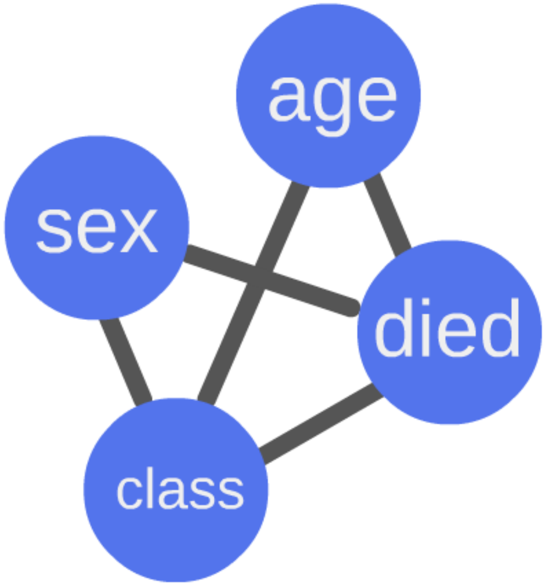

edge addition	AIC
sex-age	72

=> stop

To redo this experiment

Just 4 lines of R

```
> library(MASS)
> data(Titanic)
> independence=loglm(~Class+Sex+Survived+Age,data=Titanic)
> step(independence,scope="~.^2+.^3",direction="forward")
```

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class-sex	851
class-died	1,083
class-age	1,115
sex-age	1,236
age-died	1,240

73% of women survived vs 1% of men

Current AIC=825

edge addition	AIC
class-sex	419
class-died	650
class-age	683
sex-age	804
age-died	808

97% of the crew were men

Current AIC=419

edge addition	AIC
class-age	276
class-died	319
sex-age	397
age-died	401

2% of 1st class were children
8% of 2nd class were children
11% of 3rd class were children
0% of the crew were children

Current AIC=276

edge addition	AIC
class-died	117
age-died	267
sex-age	272

62% of the people in 1st class survived
41% of the people in 2nd class survived
25% of the people in 3rd class survived
24% of the crew survived

Current AIC=117

edge addition	AIC
age-died	70
sex-age	95

52% of children survived
31% of adults survived

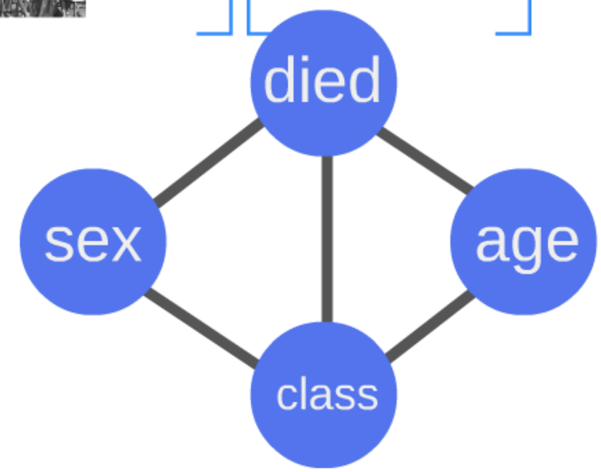
Current AIC=70

edge addition	AIC
sex-age	72

=> stop

To redo this experiment

Just 4 lines of R



To predict survival:

- Yes, age matters
- Yes, class matters
- Yes, sex matters
- Yes, class and sex together matter (eg knowing that a particular man was in 1st class or crew)
- Yes, class and age together matter (eg knowing that a particular child was in 1st or 3rd class)
- No, sex and age don't matter together for a particular class (within each class, age and sex interact with survival independently of one another)

Learning a model from data

Scoring

Bayesian approaches

Aim: Finding the model $\mathcal{M}$ that, for a dataset $\mathcal{D}$ maximizes $p(\mathcal{M}|\mathcal{D})$

$$\underbrace{p(\mathcal{M}|\mathcal{D})}_{\text{Posterior probability}} \propto \underbrace{p(\mathcal{D}|\mathcal{M})}_{\text{Likelihood}} \times \underbrace{p(\mathcal{M})}_{\text{Prior probability.}}$$

Hundreds of methods and references: BDeu, BD/BDe, MDL, NML, etc.; see details in [1,2].

[1] Koller and Friedman, Probabilistic Graphical Models, MIT Press, 2009 (esp. chapters 18 and 20)
[2] W. Buntine, A guide to the literature on learning probabilistic networks from Data, TKDE 1996.

Frequentist approaches

Avoid the definition of priors

Also hundreds of methods and approaches using statistical tests (eg Chi-squared, likelihood-ratio tests).

→ See details in [1,2,3]

$$P(\mathbf{x}) = \lim_{n_t \to \infty} \frac{n_{\mathbf{x}}}{n_t}$$

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Search

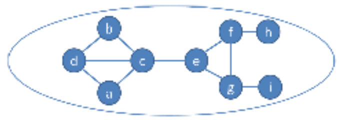
Traditional algorithms:

- local search (eg greedy, backward)
- simulated annealing
- genetic algorithms
- MCMC/Gibbs
- etc.

Note:

- BN: scores also require an order on the variables

Scoring



Data 

→ Scoring model → 32.5

Bayesian approaches

Aim: Finding the model $\mathcal{M}$ that, for a dataset $\mathcal{D}$ maximizes $p(\mathcal{M}|\mathcal{D})$

$$\begin{array}{ccccc} p(\mathcal{M}|\mathcal{D}) & \propto & p(\mathcal{D}|\mathcal{M}) & \times & p(\mathcal{M}) \\ \text{Posterior probability} & \propto & \text{Likelihood} & \times & \text{Prior probability.} \end{array}$$

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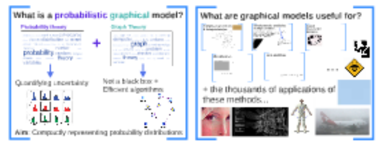
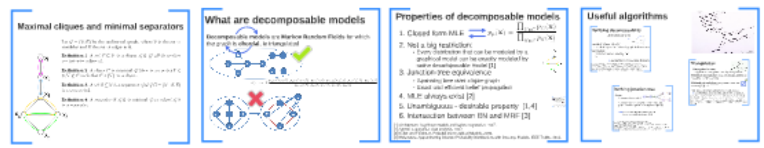
Scalable learning of graphical models

Introduction - Motivation



Graphical models 101

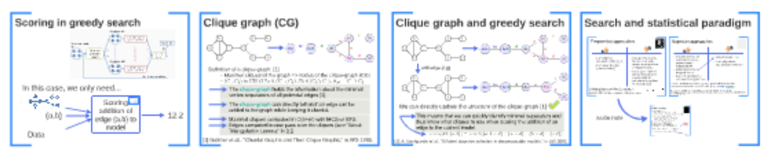
Graph theory



Evaluation - Scoring

Break

Efficient search

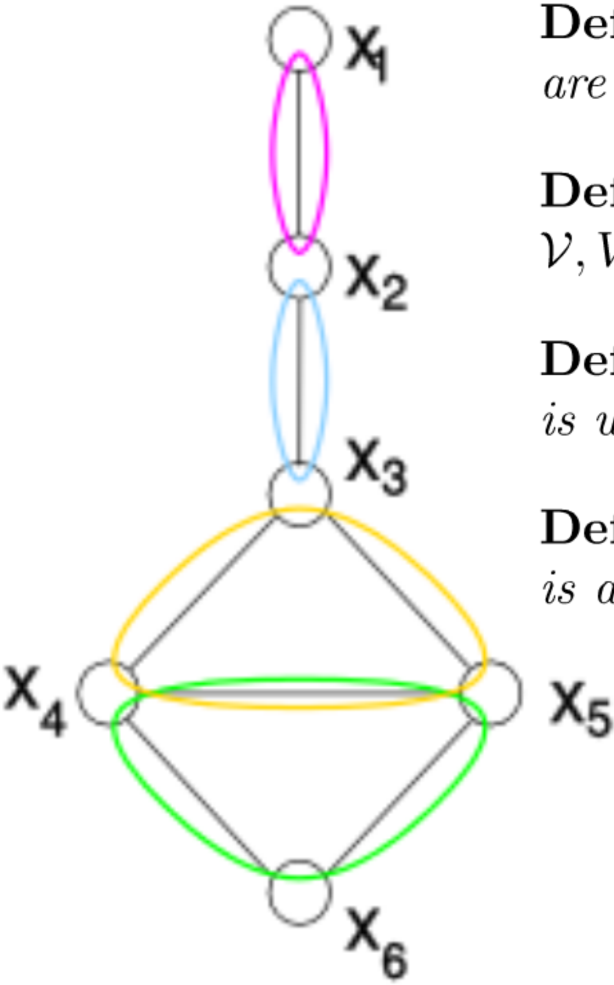


The nitty-gritty

Use cases

Wrapping up!

Maximal cliques and minimal separators



Let $\mathcal{G} = (\mathcal{V}, E)$ be the undirected graph, where $\mathcal{V}$ is the set of variables and E the set of edges in $\mathcal{G}$.

Definition 1 *A set $C \subseteq \mathcal{V}$ is a* clique *of $\mathcal{G}$ iff all its vertices are pairwise adjacent.*

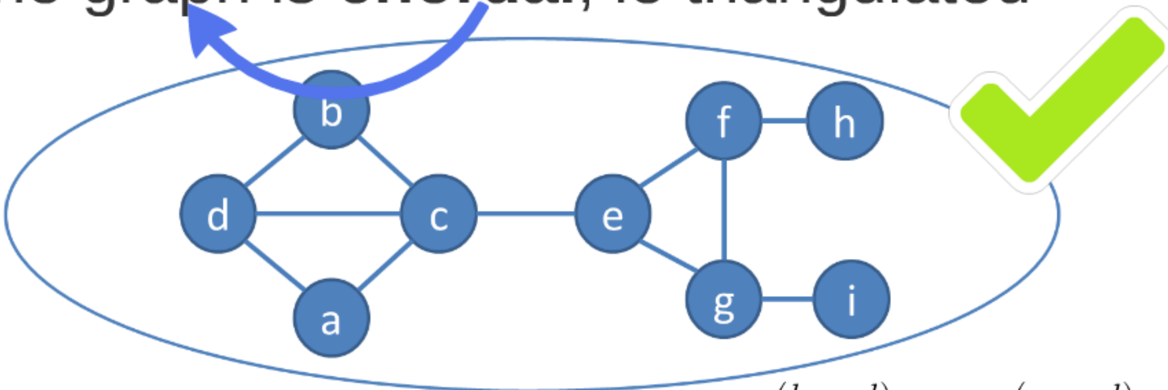
Definition 2 *A clique C is* maximal *iff there is no vertex $V \in \mathcal{V}, V \notin C$ such that $C \cup \{V\}$ is a clique.*

Definition 3 *A set $S \subseteq \mathcal{V}$ is a* separator *of $\mathcal{G}$ if $G = (\mathcal{V} - S, E)$ is unconnected.*

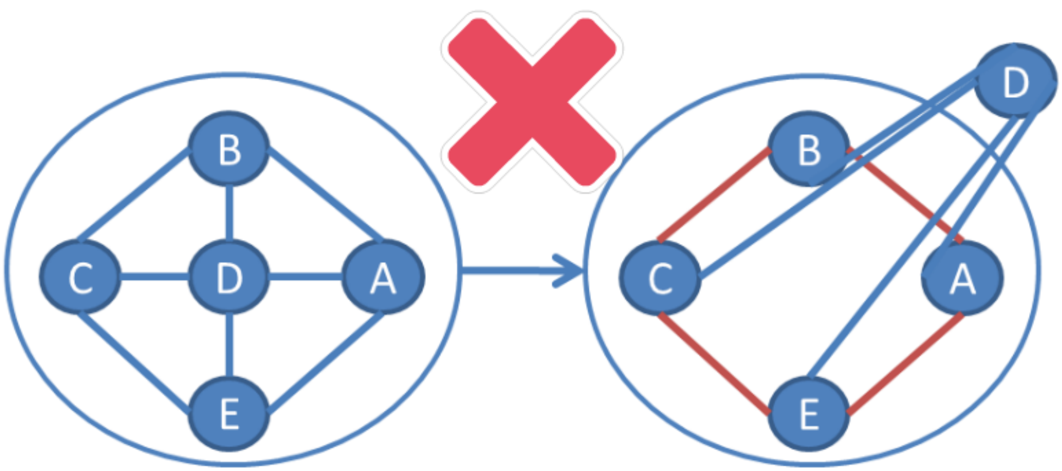
Definition 4 *A separator S of $\mathcal{G}$ is* minimal *if no subset of S is a separator.*

What are decomposable models

Decomposable models are **Markov Random Fields** for which the graph is **chordal**, ie triangulated



$$E_{a,\cdots,i} = N \cdot \frac{p_{BCD}(b,c,d)\cdot p_{ACD}(a,c,d)\cdot p_{CE}(c,e)\cdot p_{EFG}(e,f,g)\cdot p_{FH}(f,h)\cdot p_{GI}(g,i)}{p_{CD}(c,d)\cdot p_{C}(c)\cdot p_{E}(e)\cdot p_{F}(f)\cdot p_{G}(g)}$$

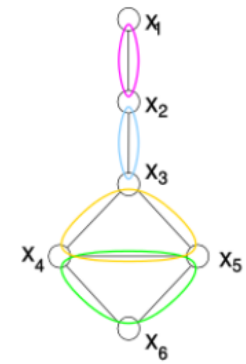


Properties of decomposable models

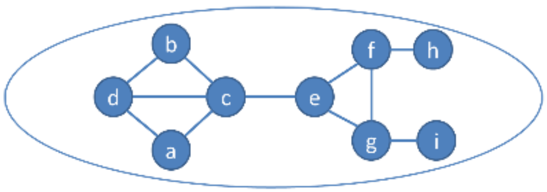
1. Closed form MLE

$$p_\mu(\mathbf{x}) = \frac{\prod_{C \in \mathcal{C}} p_C(\mathbf{x})}{\prod_{S \in \mathcal{S}} p_S(\mathbf{x})}$$

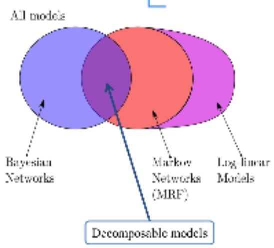
2. Not a big restriction:
   - Every distribution that can be modeled by a graphical model can be exactly modeled by some decomposable model [1]



3. Junction-tree equivalence
   - Spanning tree over clique-graph
   - Exact and efficient belief propagation



4. MLE always exist [2]
5. Unambiguous - desirable property [1,4]
6. Intersection between BN and MRF [3]



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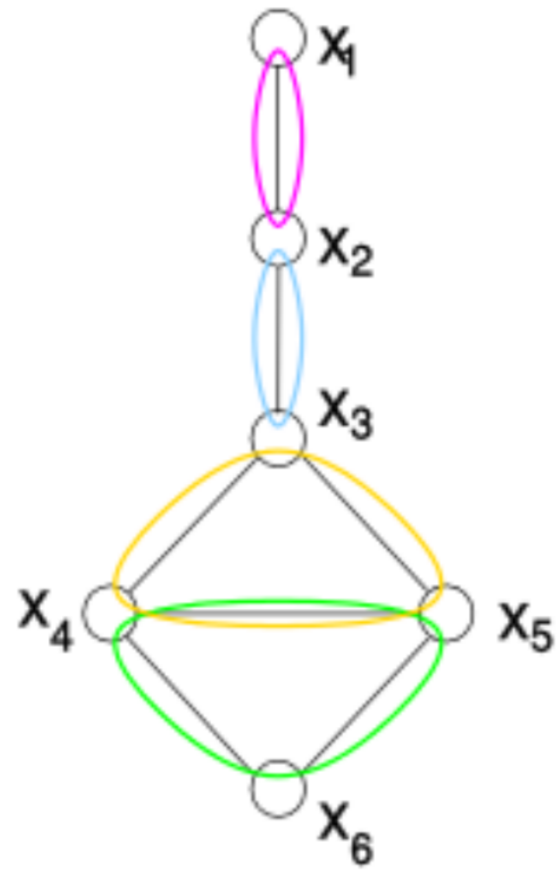

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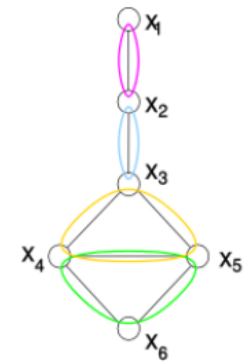

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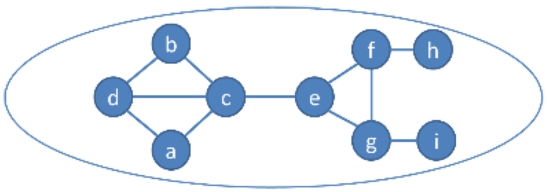

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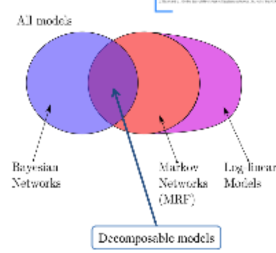

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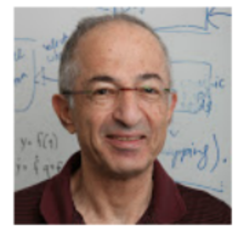
Unambiguous - interpretability

Francesco M. Malvestuto showed in [1] that:

"Since the relations of conditional independence can be treated in an axiomatic way and the associated formal system can be used as the inference engine of a common sense logic for reasoning about relevance relations, ***decomposability is a desirable quality fo belief networks****."*

This mainly comes from the fact that a chordal graph is an acyclic hypergraph (see Theorem 3.4 in [2]), which gives decomposable models the Markov property.



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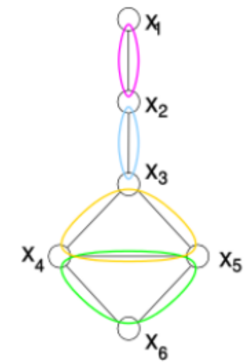

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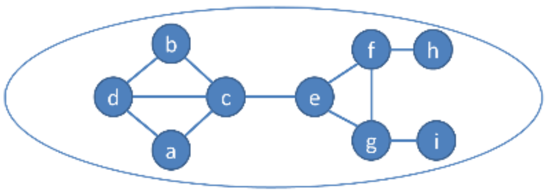

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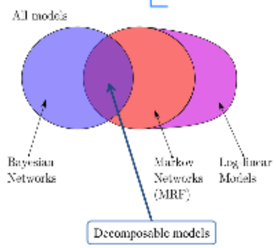

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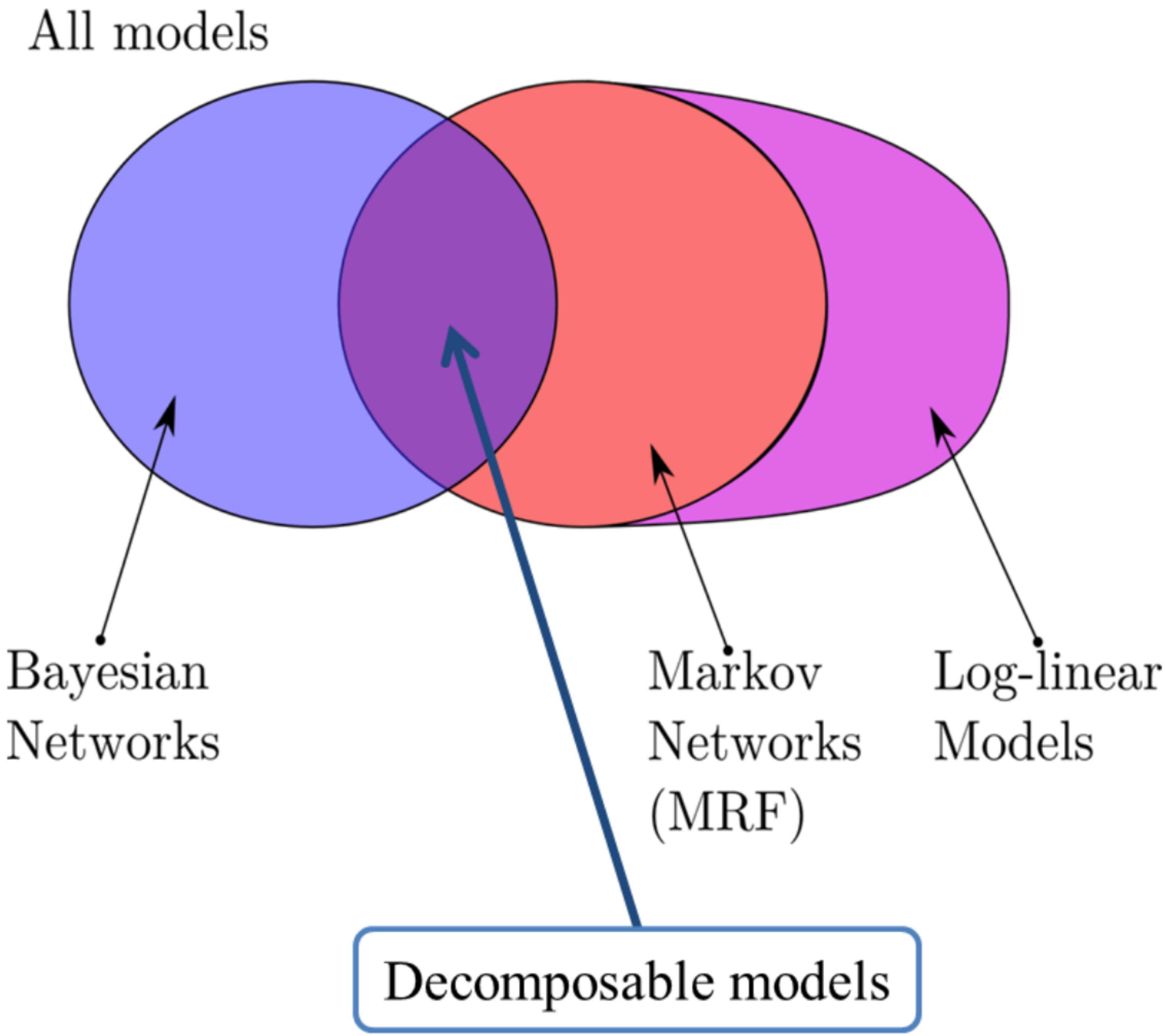
All models
Bayesian Networks
Markov Networks (MRF)
Log-linear Models
Decomposable models

Properties of decomposable models

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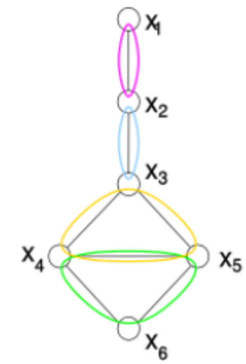

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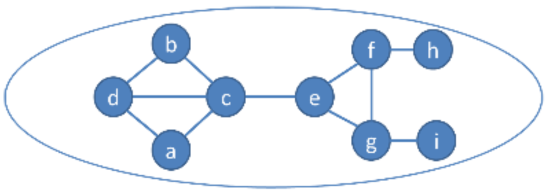

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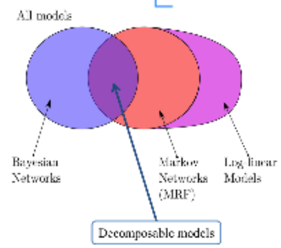

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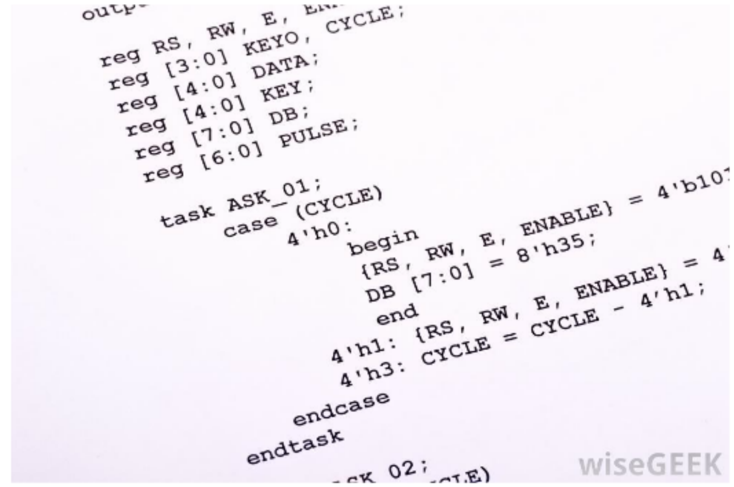
Useful algorithms



Verifying decomposability

Elimination game [1]

Lex-BFS [2] and MCS [3]

- can find a peo for a chordal graph in linear time

Verification:

1. find an vertex ordering α
2. $\text{chordal} \leftarrow (EliminationGame(G, \alpha) == G)$

→ Recognition in linear time **O(n+m)**

[1] D.R. Fulkerson et al. Incidence matrices and interval graphs, Pacific J. Math. 1965.
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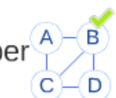
Triangulation

Triangulation is easy

- eg Elimination game actually triangulates

Minimum triangulation = as few edges added as possible => NP-hard [1]

Minimal triangulation = only one chord per square [2,3] => $o(n^{2.376})$



Heuristics and simplifications for restricted classes

→ bounded degree, perfect, trapezoid, AT-free, planar, ...

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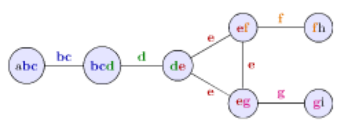
Deriving junction-tree

Steps:

1. compute clique graph [1]



2. compute a maximum spanning tree on the clique graph - Kruskal's algorithm with negative weights [2]

→ Linear-time algorithms exist based on Maximum Cardinality Search [1,3]

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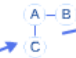
Verifying decomposability

Elimination game [1]

Algorithm Elimination Game
Input: A graph $G = (V, E)$ and an ordering $\alpha = (v_1, ..., v_n)$ of V.
Output: The filled graph G^+_α.
begin
$G^0 = G$;
for $i = 1$ **to** n **do**
Let $F^i = D_{G^{i-1}}(v_i)$;
Obtain G^i by adding the edges in F^i to G^{i-1} and removing v_i;
$G^+_\alpha = (V, E \cup \bigcup_{i=1}^{n} F^i)$;
end

(c) Heggernes, 2006



$\{b, c\} \in D(a)$

With $\alpha = (b, a, c)$
=> no edges added
=> chordal
(=> α is a perfect elimination ordering)

1965

→ Problem: finding α

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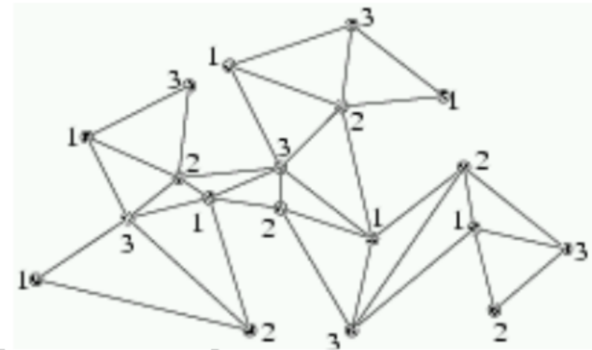

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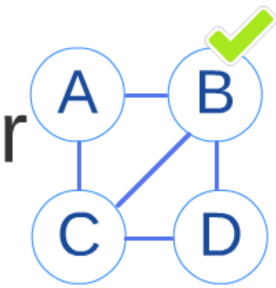

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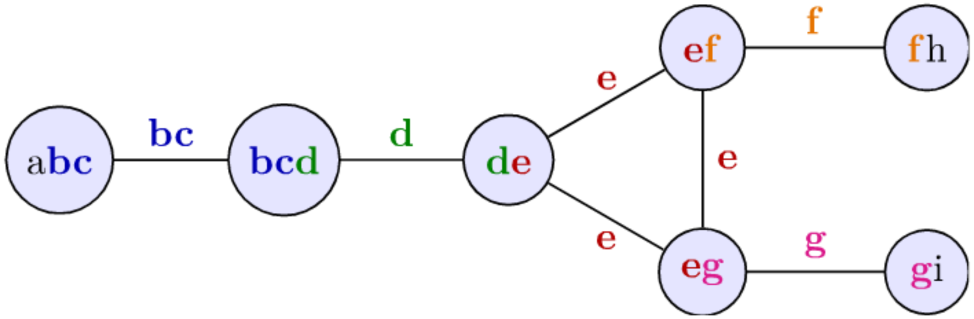

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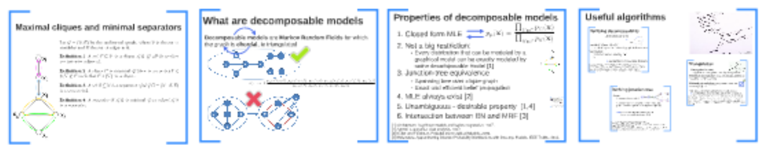
Scalable learning of graphical models

Introduction - Motivation

Graphical models 101

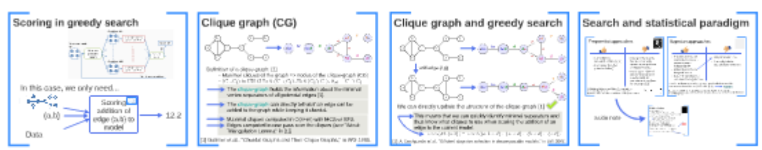
Graph theory



Evaluation - Scoring

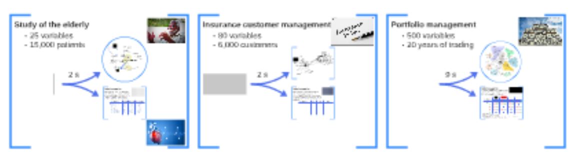
Break

Efficient search



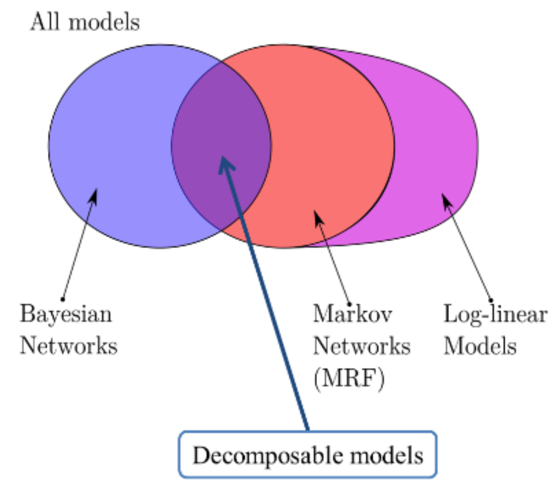
The nitty-gritty

Use cases



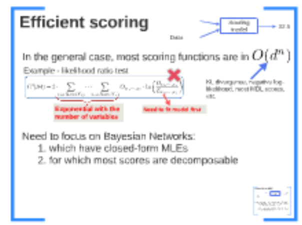
Wrapping up!

Decomposable models are essential for scalability, because...

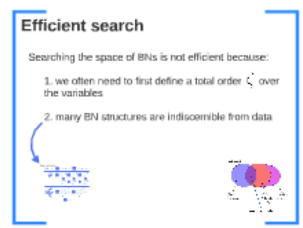


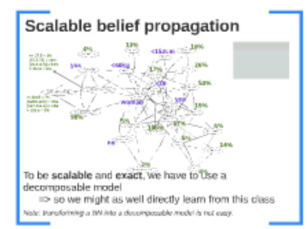
... we need:

1. scalable scoring



2. efficient search

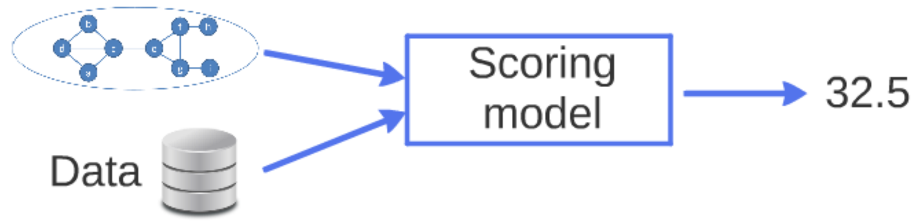




3. scalable belief propagation

+ all the results we will show here

Efficient scoring



In the general case, most scoring functions are in $O(d^n)$

KL divergence, negative log-likelihood, most MDL scores, etc.

Example - likelihood ratio test

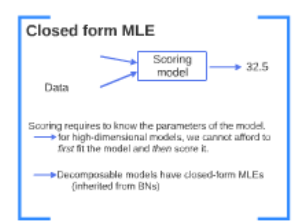
$$G^2(\mathcal{M}) = 2 \cdot \sum_{x_1 \in Dom(X_1)} \cdots \sum_{x_n \in Dom(X_n)} O_{x_1,\cdots,x_n} \cdot \ln\left(\frac{O_{x_1,\cdots,x_n}}{E_{x_1,\cdots,x_n}}\right)$$

Exponential with the number of variables

Need to fit model *first*

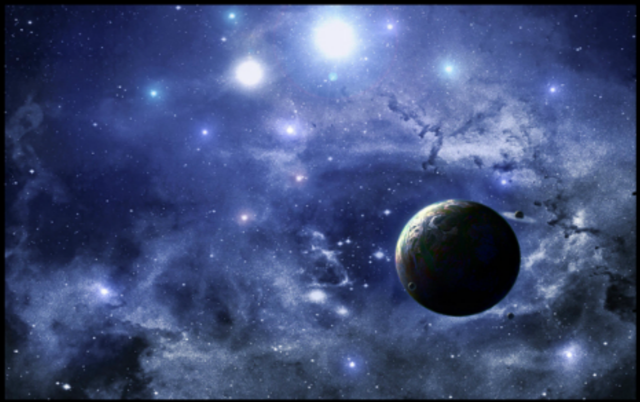
Need to focus on Bayesian Networks:

1. which have closed-form MLEs
2. for which most scores are decomposable



1,000 binary variables

$$\underbrace{10\cdots000000}_{\textbf{300}}$$ operations



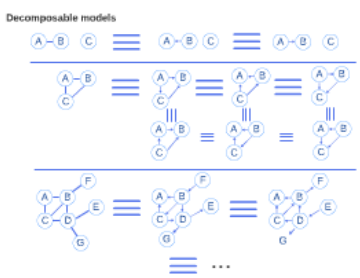
10^{82} *atoms in the observable universe...*

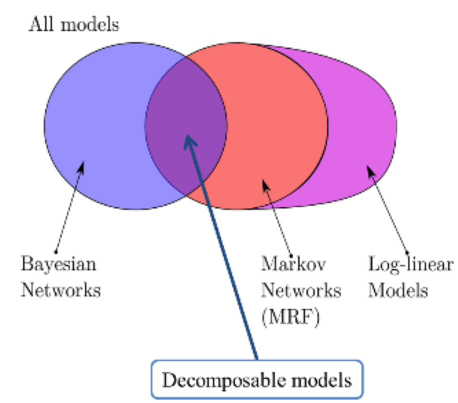
Efficient search

Searching the space of BNs is not efficient because:

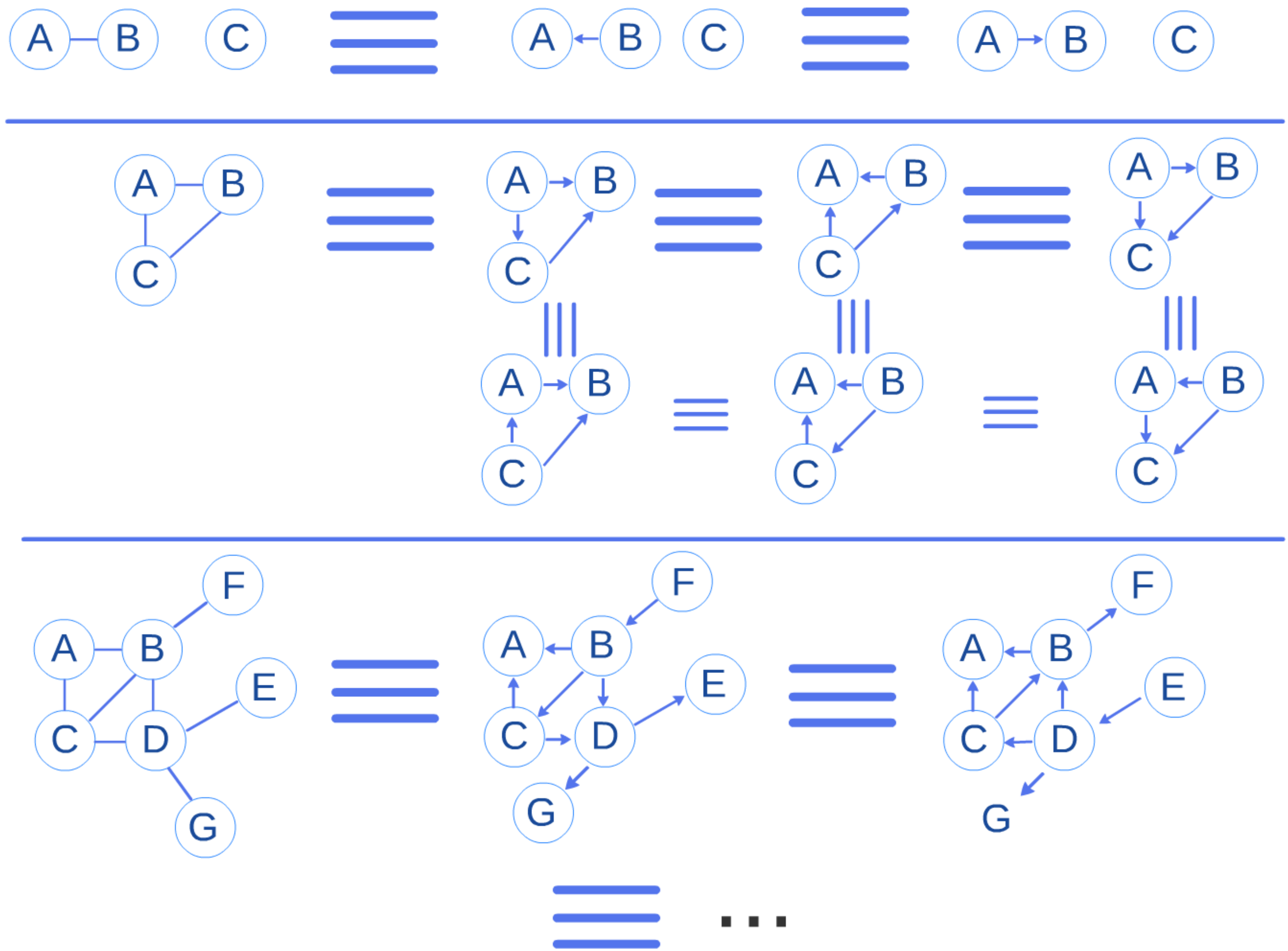
1. we often need to first define a total order ζ over the variables

2. many BN structures are indiscernible from data

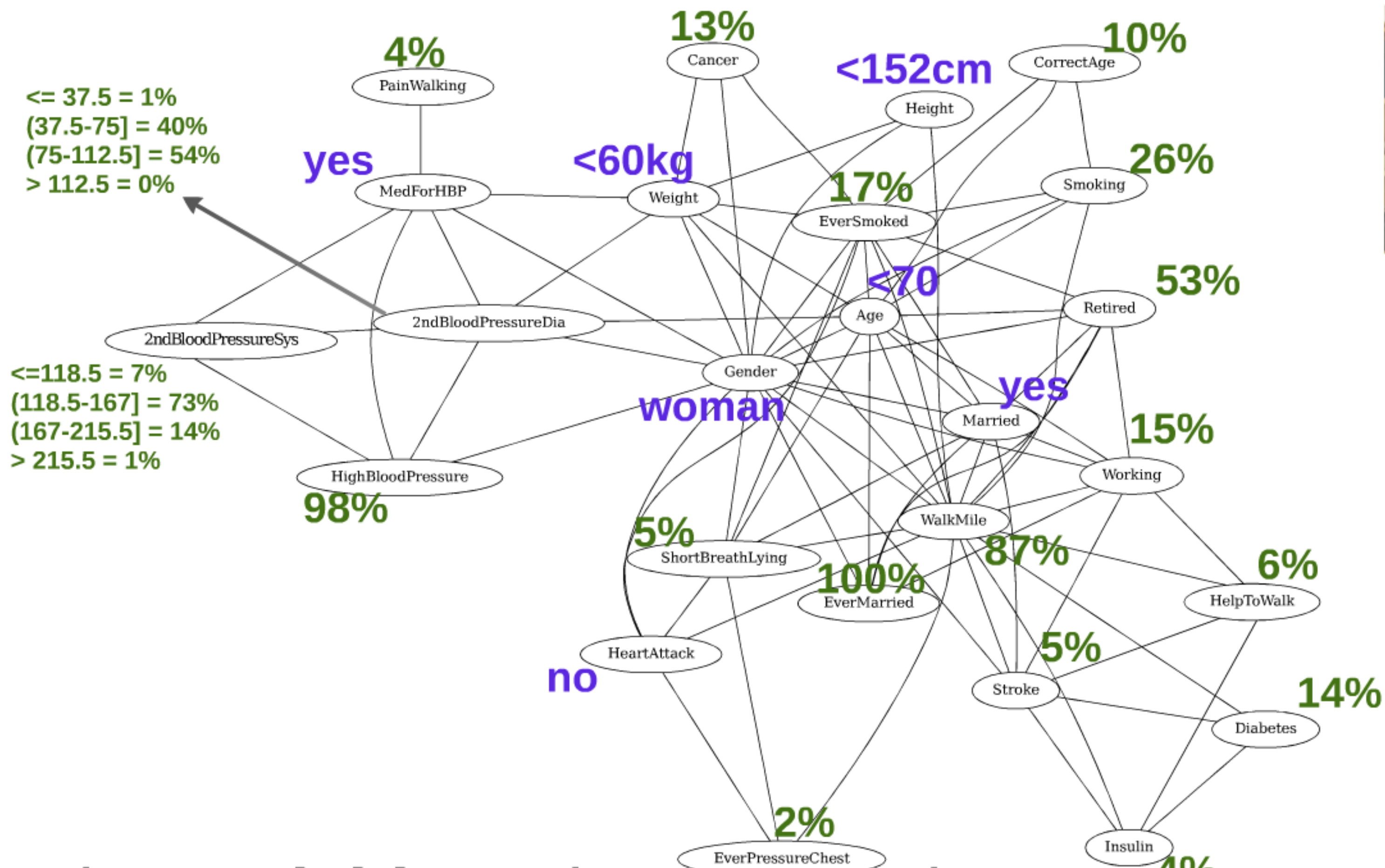




Decomposable models



Scalable belief propagation



To be **scalable** and **exact**, we have to use a decomposable model

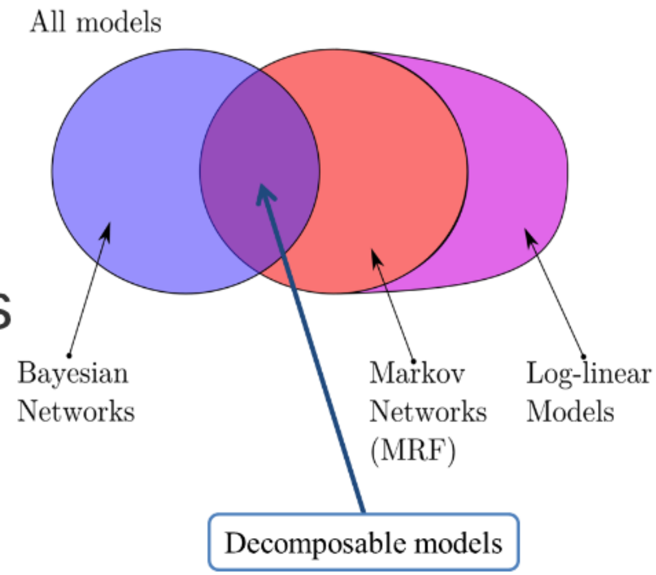
=> so we might as well directly learn from this class

Note: transforming a BN into a decomposable model is not easy.

Bottom line

A decomposable model is equivalent to:

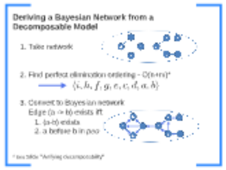
- a Markov Network
- a set of equivalent Bayesian Networks



→ Any scoring function that has been developed for **MRFs*** or for **BNs** **can be used** for decomposable models

→ MRF: direct applicability

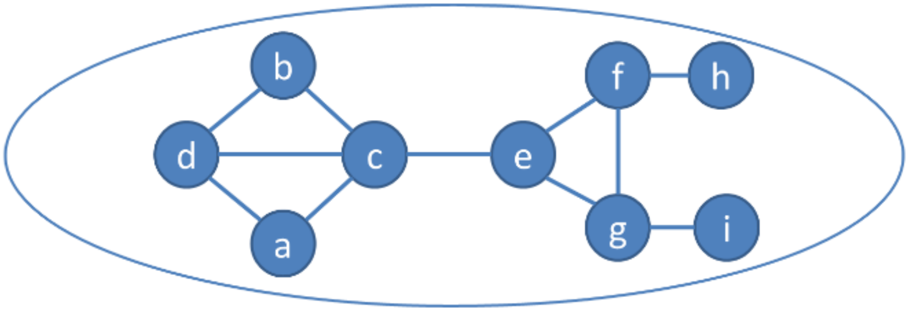
→ BN: derive and equivalent BN first and then use the score on it



* this implies metrics developed for log-linear models as well

Deriving a Bayesian Network from a Decomposable Model

1. Take network



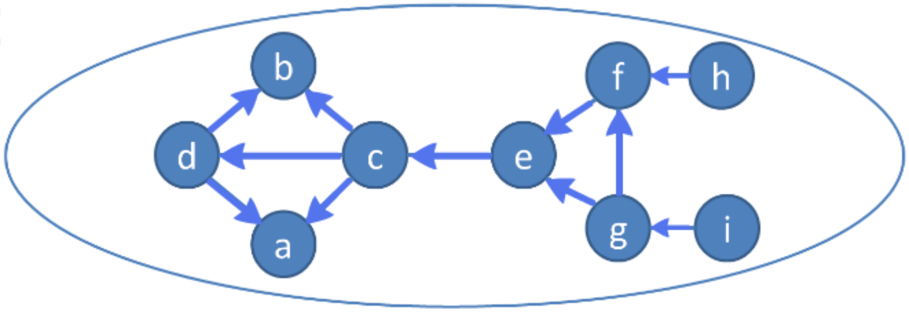
2. Find perfect elimination ordering - O(n+m)*

$$\longrightarrow \langle i, h, f, g, e, c, d, a, b \rangle$$

3. Convert to Bayesian network

Edge (a -> b) exists iff:

1. (a-b) exists
2. a before b in *peo*



* see Slide "Verifying decomposability"

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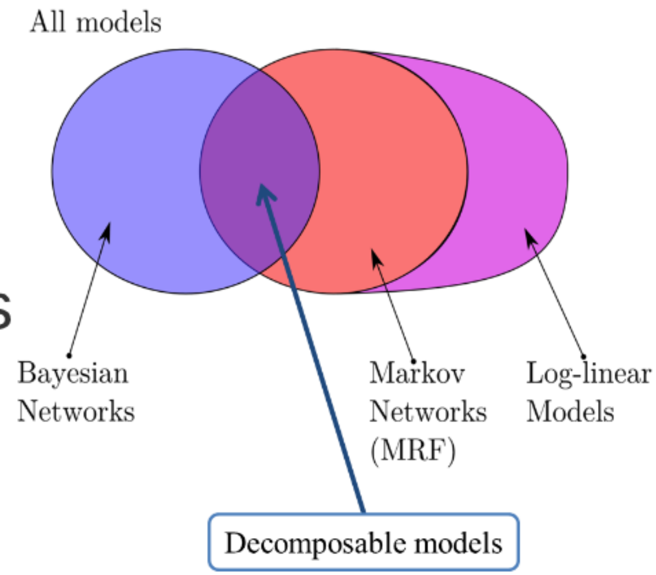

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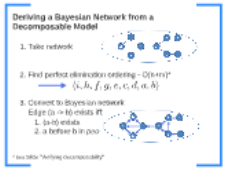

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Most scores are scalable

Entropy [1] ✔

$$H(M) = -\sum_{x_1 \in Dom(X_1)} \cdots \sum_{x_n \in Dom(X_n)} p_M(x_1, \cdots, x_n) \cdot \ln p_M(x_1, \cdots, x_n)$$
$$= \sum_{C \in \mathcal{C}} H(X_C) - \sum_{S \in \mathcal{S}} H(X_S)$$

→ $O(2^n) \Rightarrow O(2^k)$
where k is the size of the biggest clique

Kullback Leibler [1,2] (because is minimized when entropy is also) ✔

G-test statistic [3] ✔

MML / MDL [4,5] ✔

[1] Malvestuto, Approximating Discrete Probability Distributions with Decomp. Models, IEEE TSMC, 1991.
[2] Deshpande et al, Efficient Stepwise Selection in Decomposable Models, UAI 2001.
[3] **Petitjean**, Nicholson and **Webb**, Scaling log-linear analysis to high-dimensional data, IEEE ICDM 2013.
[4] Altmueller and Haralick, Approximating High Dimensional Probability Distributions, ICPR 2004.
[5] **Petitjean**, Allison and **Webb**, A statistically efficient and scalable method for log-linear analysis of high-dimensional data, IEEE ICDM 2014.

$$
\begin{aligned}
H(\mathcal{M}) &= -\sum_{x_1 \in Dom(X_1)} \cdots \sum_{x_n \in Dom(X_n)} p_\mu(x_1, \cdots, x_n) \cdot \ln p_\mu(x_1, \cdots, x_n) \\
&= \sum_{C \in \mathcal{C}} H(X_C) - \sum_{S \in \mathcal{S}} H(X_S)
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Scalable learning of graphical models

Introduction - Motivation

Graphical models 101

Graph theory

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Efficient search

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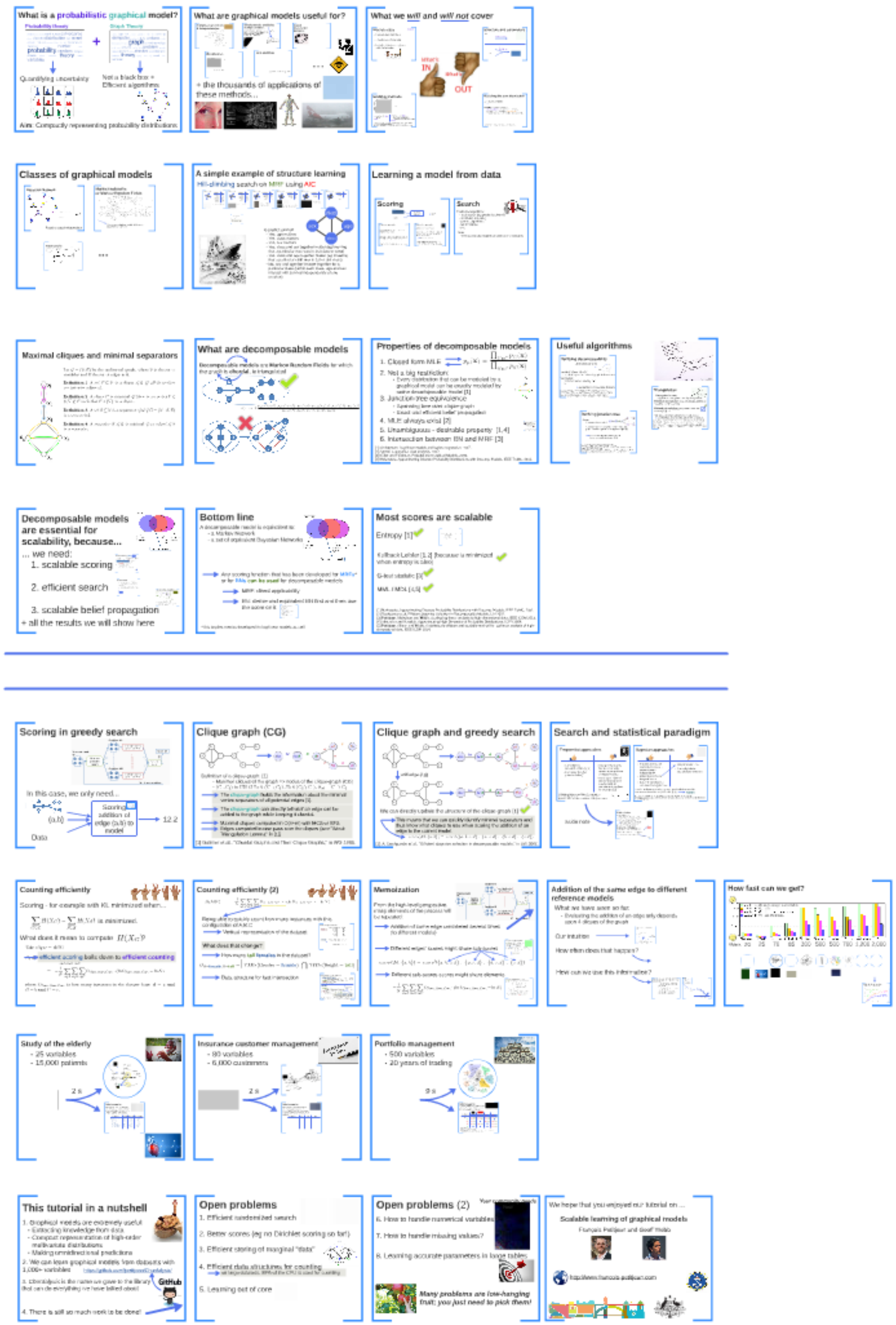
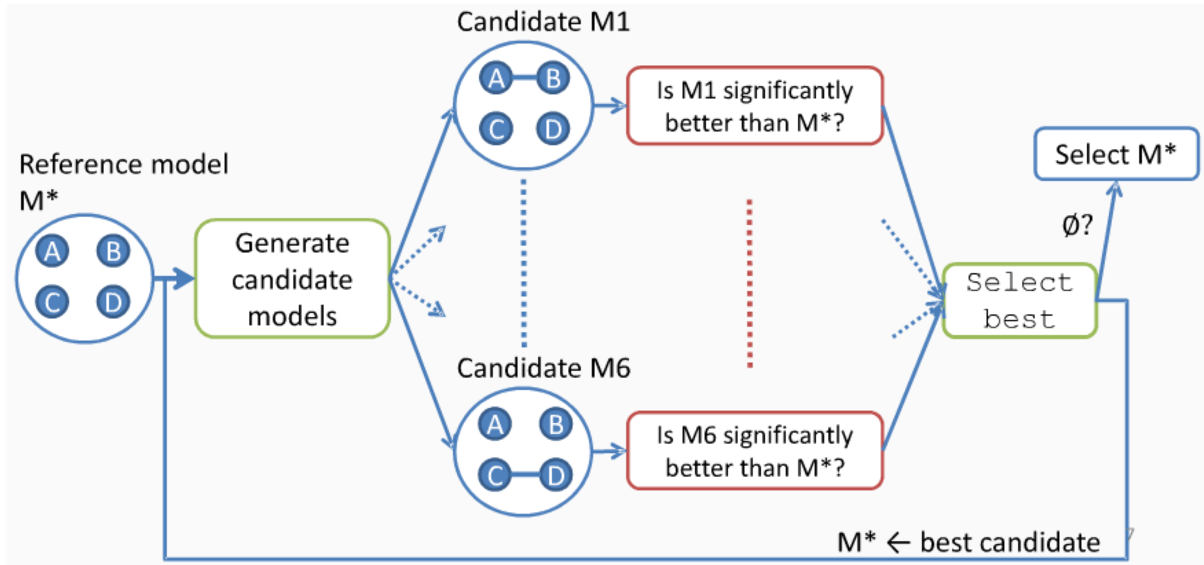
Use cases

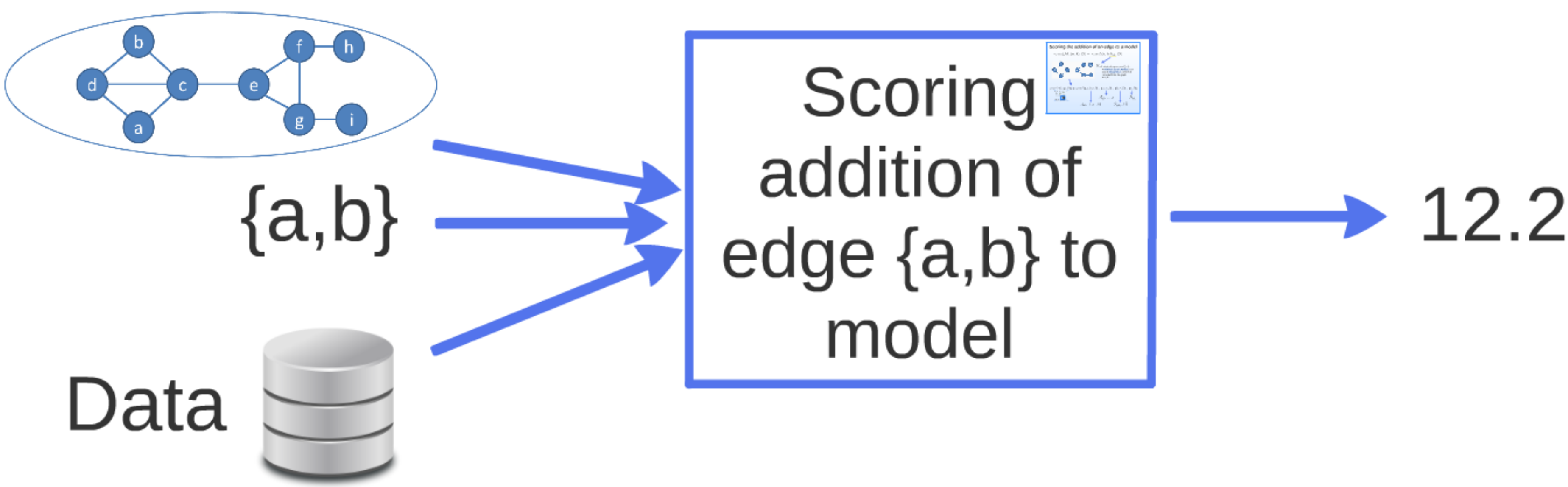
Wrapping up!



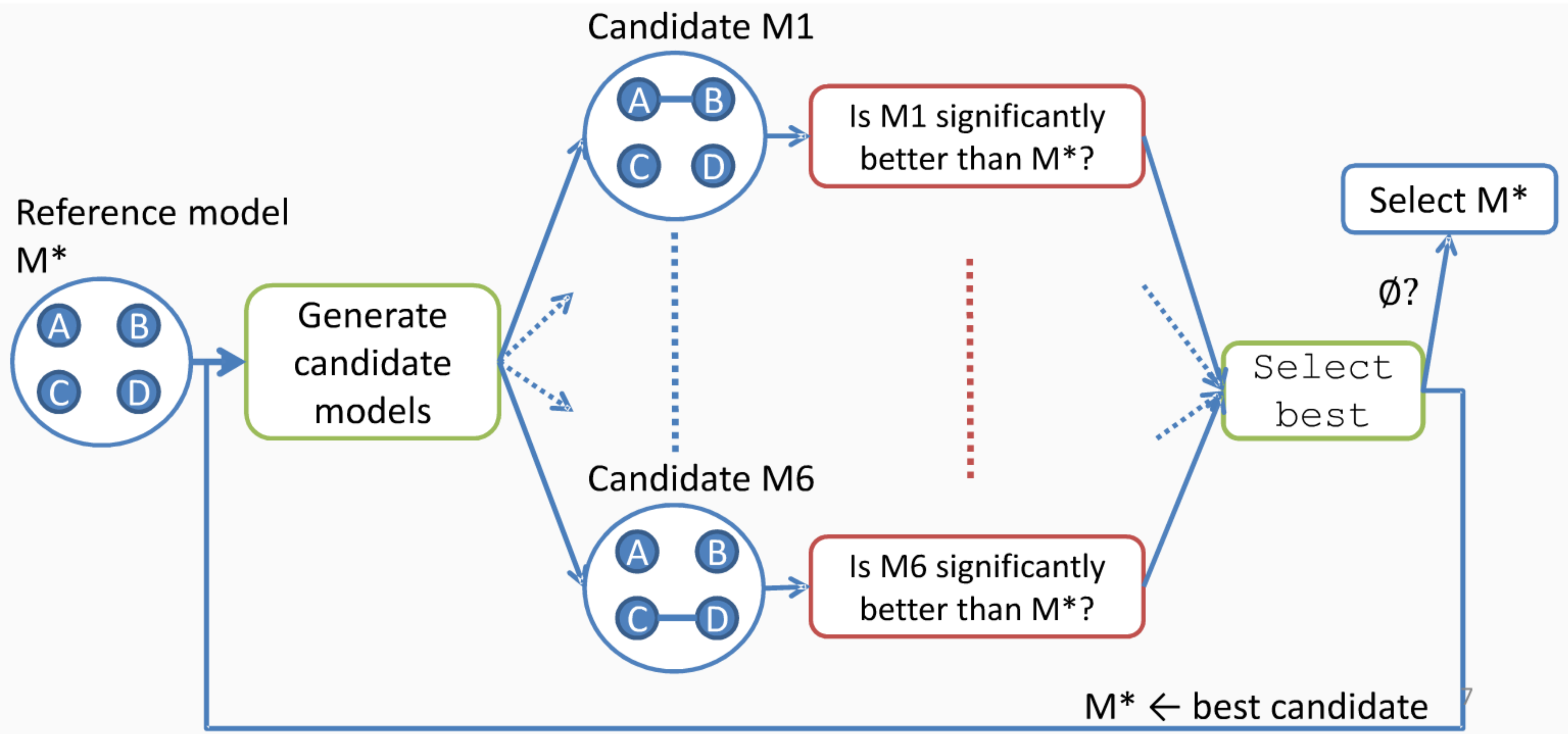
Scoring in greedy search



In this case, we only need...



greedy search



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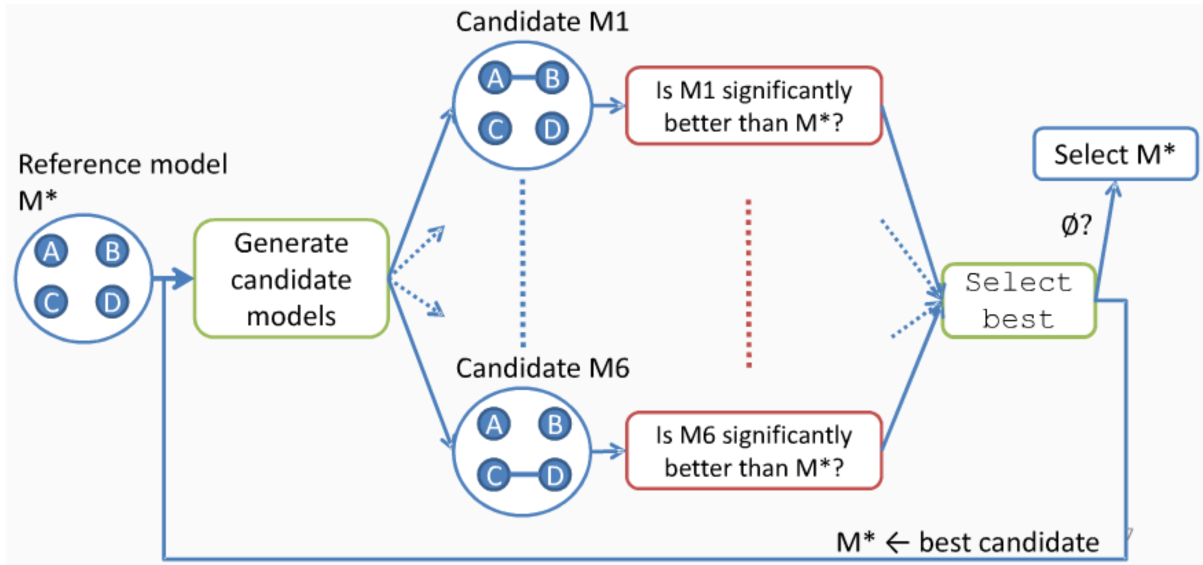
Scoring in greedy search



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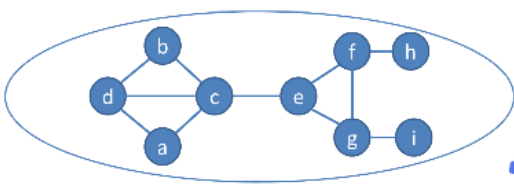


{a,b}

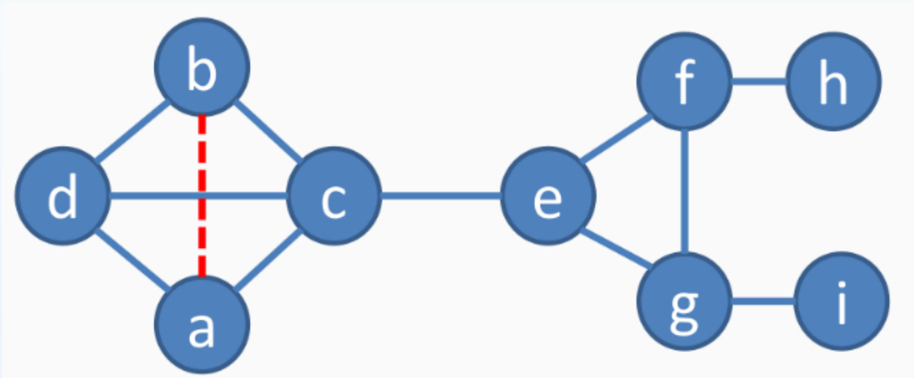
Data

Scoring addition of edge {a,b} to model

12.2

Scoring *the addition of an edge to a model*

$$score(\mathcal{M}, (a,b), \mathcal{D}) = score'(a, b, S_{ab}, \mathcal{D})$$



S_{ab} : minimal separator of (a,b)
= **minimal set of vertices** that would **disconnect** *a* from *b* if removed from the graph
= {c,d}

$$score(\mathcal{M}, \{a,b\}) = score'(\{a,b,c,d\}\ ,\ \{a,c,d\}\ ,\ \{b,c,d\}\ ,\ \{c,d\})$$

$$S_{ab} \cup a \cup b \qquad S_{ab} \cup a \qquad S_{ab} \cup b \qquad S_{ab}$$

This has been proven for different scorings

- Statistical tests (G-test) [1] ✓
- MML/MDL [2] ✓
- Entropy / KL divergence [3] ✓

[1]: F. Petitjean *et al.*, "Scaling log-linear analysis to high-dimensional data," in *ICDM 2013*.
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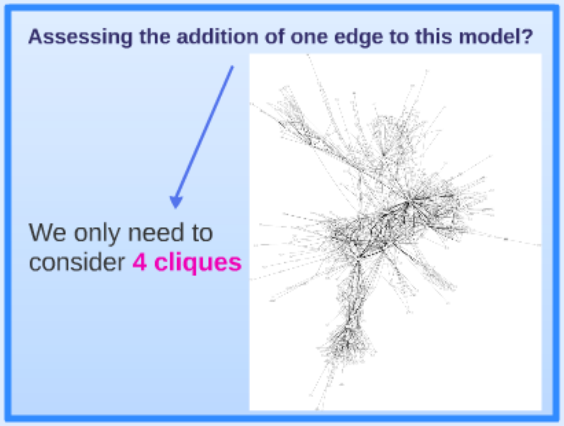

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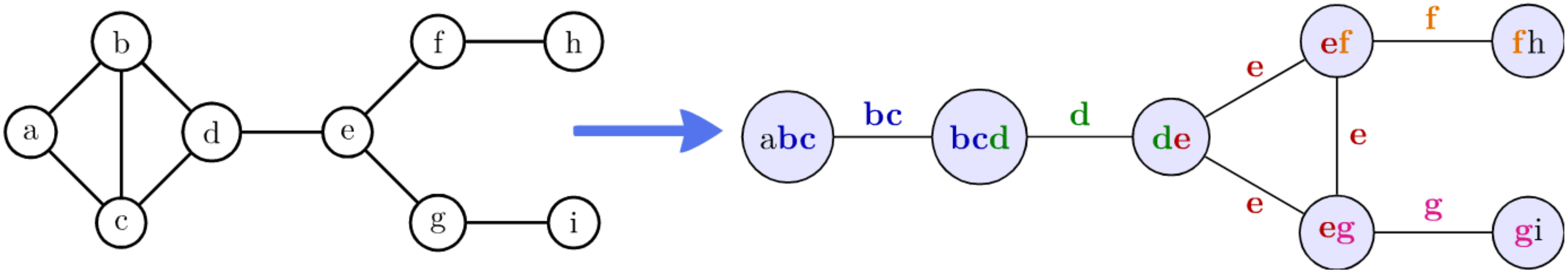
Assessing the addition of one edge to this model?

We only need to consider **4 cliques**

Clique graph (CG)



Definition of a clique-graph: [1]

- Maximal cliques of the graph => nodes of the clique-graph (CG)
- (C_1, C_2) in CG iff $\forall a \in (C_1 \setminus C_2), \forall b \in (C_2 \setminus C_1), S_{ab} = C_1 \cap C_2$

→ The **clique-graph** holds the information about the minimal vertex separators of all potential edges [1].

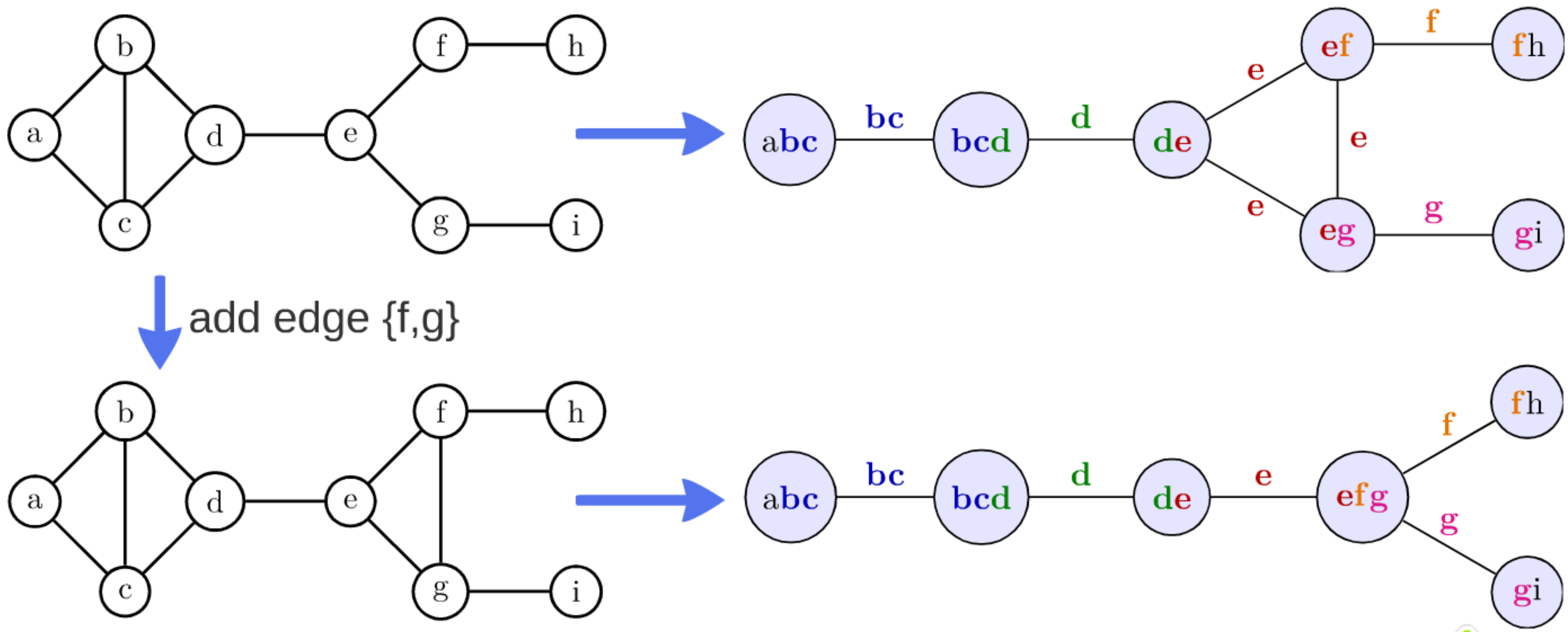
→ The **clique-graph** can directly tell us if an edge can be added to the graph while keeping it chordal.

→ Maximal cliques computed in O(n+m) with MCS or BFS.

→ Edges computed in one pass over the cliques (see "Weak Triangulation Lemma" in [1])

[1] Galinier et al., "Chordal Graphs and Their Clique Graphs," in *WG 1995*.

Clique graph and greedy search



We can directly update the structure of the clique graph [1]

This means that we can quickly identify minimal separators and thus know what cliques to use when scoring the addition of an edge to the current model.

$$score(\mathcal{M}, \{a, b\}) = score'(\{a, b, c, d\}\ ,\ \{a, c, d\}\ ,\ \{b, c, d\}\ ,\ \{c, d\})$$

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Search and statistical paradigm

Frequentist approaches

Pros:

- Currently best statistical efficiency [1]
- Parameter-free (no priors to define)

Cons:

- Only greedy search, because can only score the comparison of nested models
- Growing criticism of the community when used directly for decision making (see for example [2])

$$P(\mathbf{x}) = \lim_{n_t \to \infty} \frac{n_{\mathbf{x}}}{n_t}$$

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[2] Nuzzo, "Scientific method: Statistical errors", Nature 2014.

Bayesian approaches

Pros:

- Randomized search available, because it scores models independently
- Makes it possible to integrate priors
- Easier integration in a decision making process

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- Not parameter-free
- Currently inferior statistical performance*

* So far, no Bayesian scoring has been specifically developed for decomposable models (only MDL/MML [1,2]) → **Open**

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aside note

Multiple testing

Frequentist approaches:

1. Choose a significance level α $(\text{eg} = 0.01)$
2. Assess probability p of observing data given null hypothesis
3. If $p < \alpha$ then reject null hypothesis

→ This guarantees that the chance of falsely rejecting the null hypothesis is less than α

Why do we need multiple testing corrections?

$p(\text{making an error in 1 test} \mid \text{null is true}) = \alpha$
$p(\text{not making an error in 1 test} \mid \text{null is true}) = 1 - \alpha$
$p(\text{not making an error in T tests} \mid \text{null is true}) = (1 - \alpha)^T$
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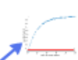

Standard solution: choose $\alpha' = \frac{\alpha}{T}$ (Bonferroni)

But, for model selection, we do not know T → solutions

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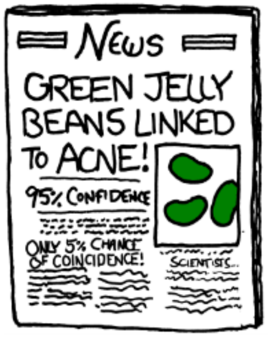

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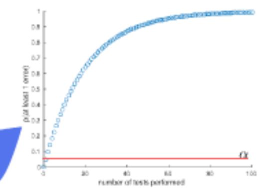

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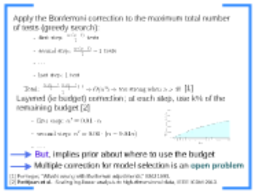
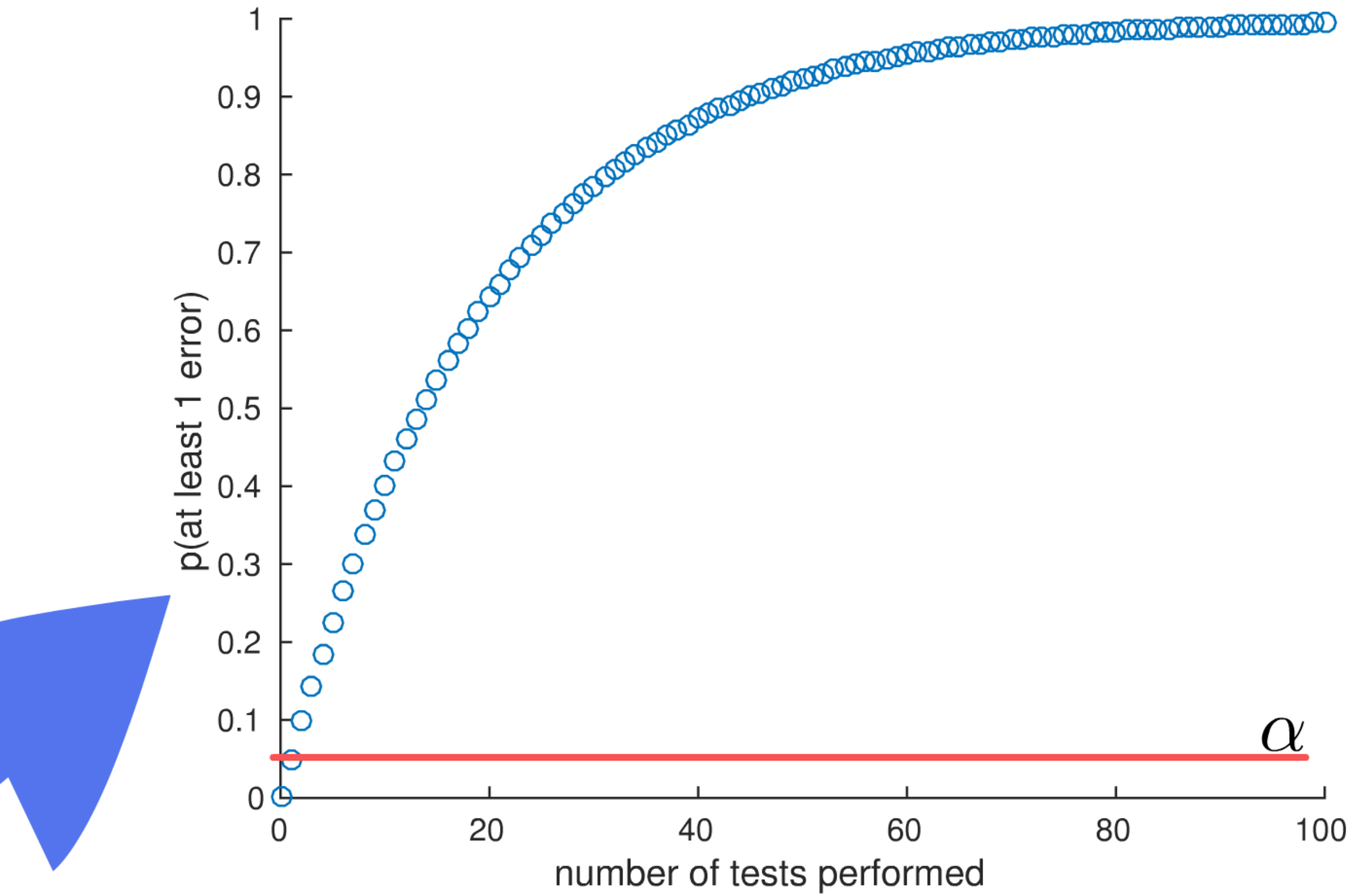
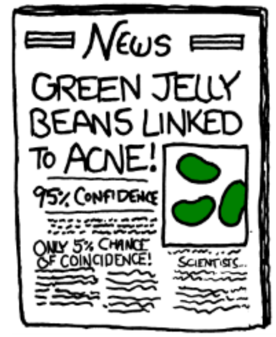




1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
p(at least 1 error)
0
20
40
60
80
100
number of tests performed
α

Multiple testing



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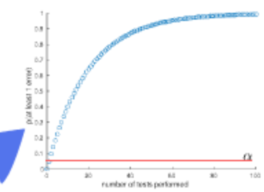

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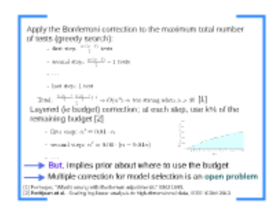


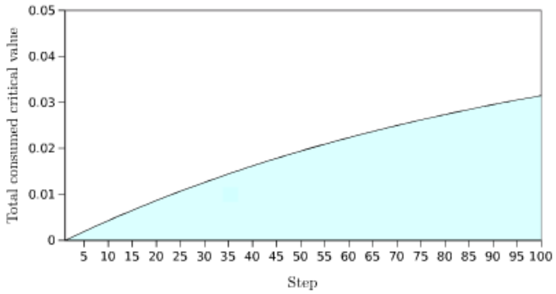
Apply the Bonferroni correction to the maximum total number of tests (greedy search):

- first step: $\frac{n \cdot (n-1)}{2}$ tests
- second step: $\frac{n \cdot (n-1)}{2} - 1$ tests
- $\ldots$
- last step: 1 test

Total: $\frac{\frac{n \cdot (n-1)}{2} \cdot \frac{n \cdot (n-1)}{2} + 1}{2} \Rightarrow O(n^4) \Rightarrow$ too strong when $n > 30$ [1]

Layered (ie budget) correction; at each step, use k% of the remaining budget [2]

- first step: $\alpha' = 0.01 \cdot \alpha$
- second step: $\alpha' = 0.01 \cdot (\alpha - 0.01\alpha)$
- $\ldots$



→ **But**, implies prior about where to use the budget

→ Multiple correction for model selection is an **open problem**

[1] Perneger, "What's wrong with Bonferroni adjustments," BMJ 1998.
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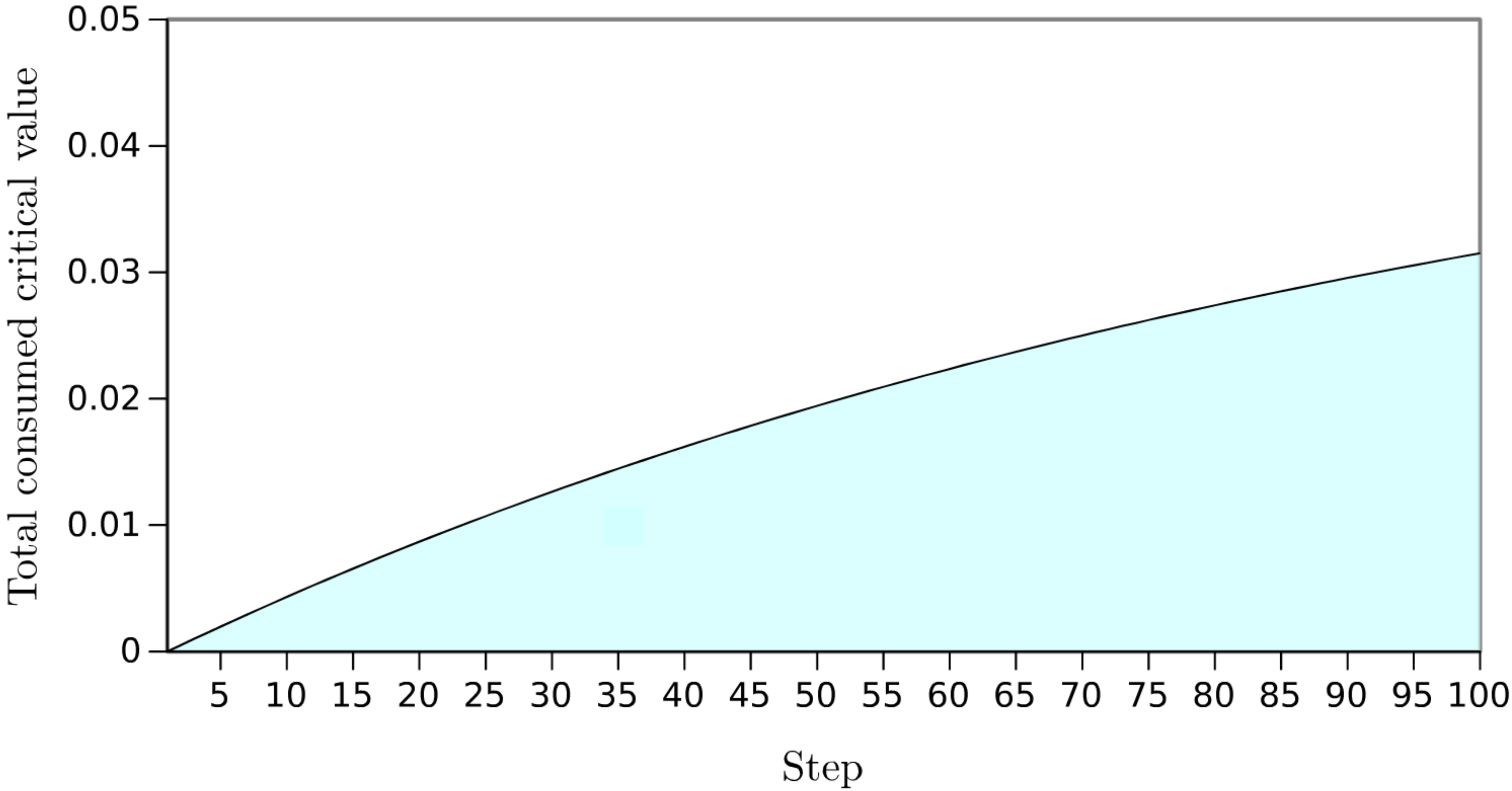

0.05
0.04
0.03
0.02
0.01
0
Total consumed critical value
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
Step

Apply the Bonferroni correction to the maximum total number of tests (greedy search):

- first step: $\frac{n\cdot(n-1)}{2}$ tests
- second step: $\frac{n\cdot(n-1)}{2} - 1$ tests
- $\ldots$
- last step: 1 test

Total: $\frac{\frac{n\cdot(n-1)}{2}\cdot\frac{n\cdot(n-1)}{2}+1}{2} \Rightarrow O(n^4) \Rightarrow$ too strong when $n > 30$ [1]

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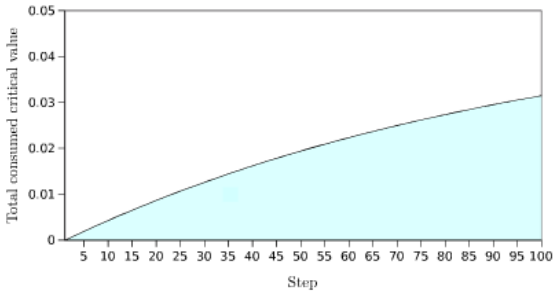

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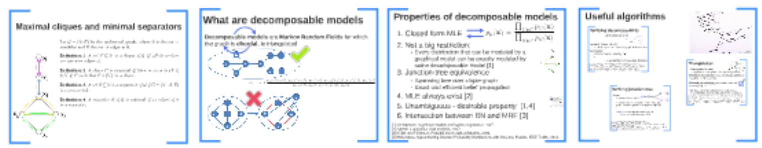
Scalable learning of graphical models

Introduction - Motivation

Graphical models 101

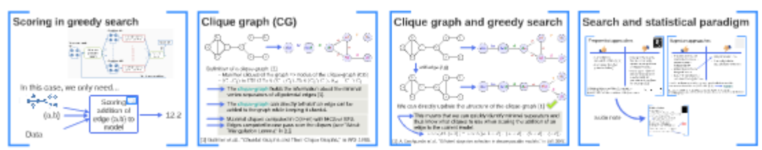
Graph theory



Evaluation - Scoring

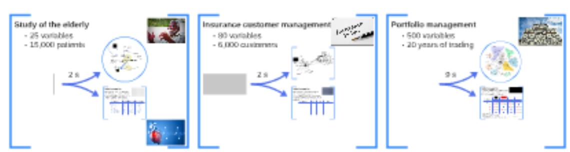
Break

Efficient search



The nitty-gritty

Use cases



Wrapping up!

Counting efficiently

Scoring - for example with KL minimized when...

$$\sum_{C \in \mathcal{C}} H(X_C) - \sum_{S \in \mathcal{S}} H(X_S)$$ is minimized.

What does it mean to compute $H(X_C)$?

Take $clique = ABC$:

→ efficient scoring boils down to **efficient counting**

$$= -\frac{1}{N} \sum_{a \in A} \sum_{b \in B} \sum_{c \in C} O_{A=a,B=b,C=c} \cdot (\ln O_{A=a,B=b,C=c} - \ln N)$$

where $O_{A=a,B=b,C=c}$ is how many instances in the dataset have $A = a$ **and** $B = b$ **and** $C = c$.

Counting efficiently (2)

$$H(ABC) = -\frac{1}{N}\sum_{a\in A}\sum_{b\in B}\sum_{c\in C} O_{A=a,B=b,C=c}\cdot(\ln O_{A=a,B=b,C=c} - \ln N)$$

Being able to quickly count how many instances with this configuration of A,B,C

→ Vertical representation of the dataset

Vertical representation

Horizontal

TID	Gender	Age	Height
1	female	60+	tall
2	female	10-20	short
3	male	40-50	tall
⋮	⋮	⋮	⋮
14,329	female	10-20	tall
14,330	male	60+	short

Vertical →

$$\begin{aligned} TIDs(\text{Gender} = \text{female}) &= \{1, 2, \cdots, 14329\} \\ TIDs(\text{Gender} = \text{male}) &= \{3, \cdots, 14330\} \\ &\vdots \\ TIDs(\text{Height} = \text{tall}) &= \{1, 3, \cdots, 14329\} \end{aligned}$$

What does that change?

→ How many **tall females** in the dataset?

$$O_{\text{G=female,H=tall}} = \left| TIDs\,(\text{Gender} = \textbf{female}) \bigcap TIDs\,(\text{Height} = \textbf{tall}) \right|$$

→ Data structure for fast intersection

Data structures for TID sets

Sorted sets of integers

$$TIDs(\text{Gender} = \text{female}) = \{1, 2, \cdots, 14329\}$$

Advantage: intersection in O(size of the largest TID set)
Drawback: storage (N x 32bits)
=> Good for sparse data

Bitmaps →

Advantages:
- intersection time independent of data sparsity
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... see also compressed bitmaps
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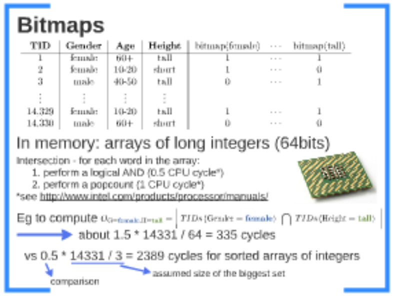

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Bitmaps

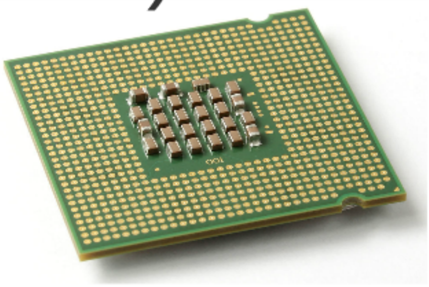
TID	Gender	Age	Height	bitmap(female)	⋯	bitmap(tall)
1	female	60+	tall	1	⋯	1
2	female	10-20	short	1	⋯	0
3	male	40-50	tall	0	⋯	1
⋮	⋮	⋮	⋮			
14,329	female	10-20	tall	1	⋯	1
14,330	male	60+	short	0	⋯	0

In memory: arrays of long integers (64bits)

Intersection - for each word in the array:

1. perform a logical AND (0.5 CPU cycle*)
2. perform a popcount (1 CPU cycle*)

*see http://www.intel.com/products/processor/manuals/



Eg to compute $o_{\text{G=female,H=tall}} = \left| TIDs\,(\text{Gender} = \textbf{female}) \bigcap TIDs\,(\text{Height} = \textbf{tall}) \right|$

→ about 1.5 * 14331 / 64 = 335 cycles

vs 0.5 * 14331 / 3 = 2389 cycles for sorted arrays of integers

comparison

assumed size of the biggest set

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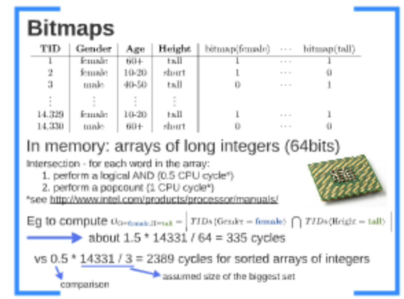

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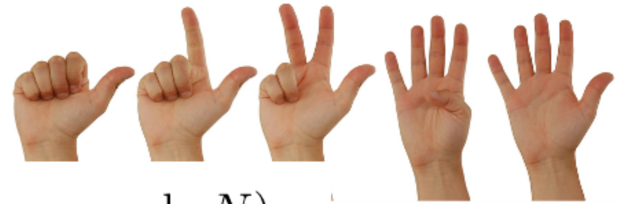

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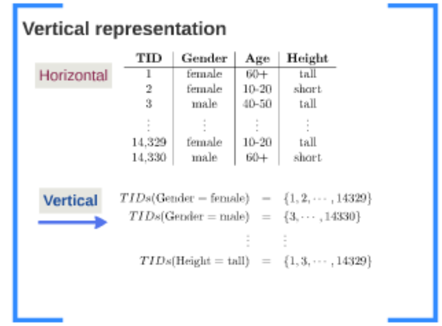

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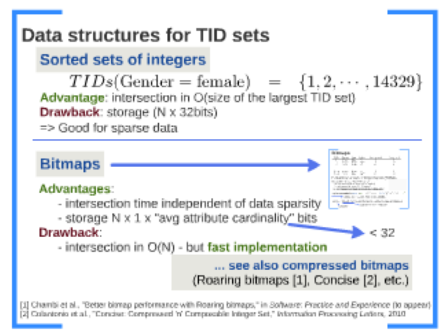

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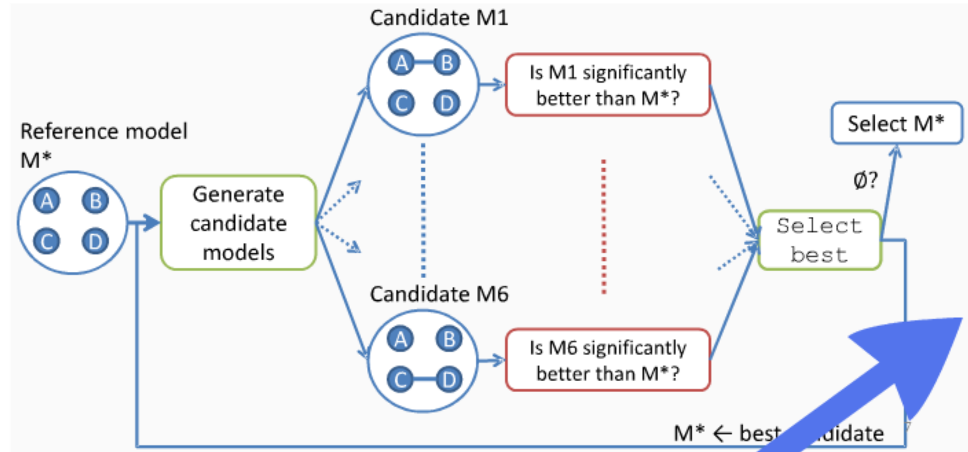
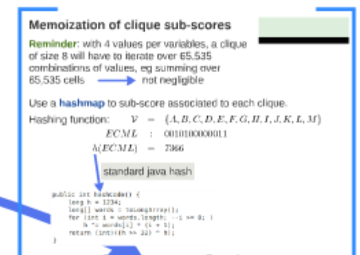
Memoization

From the high-level perspective, many elements of the process will be repeated:



→ Addition of same edge considered several times (to different models)

→ Different edges' scores might share sub-scores



Memoization of clique sub-scores

Reminder: with 4 values per variables, a clique of size 8 will have to iterate over 65,535 combinations of values, eg summing over 65,535 cells → not negligible

Use a hashmap to sub-score associated to each clique.

Hashing function: $V = (A, B, C, D, E, F, G, H, I, J, K, L, M)$

$ECML : 0010100000011$

$h(ECML) = 7366$

standard java hash

```
public int hashCode() {
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    long[] words = toLongArray();
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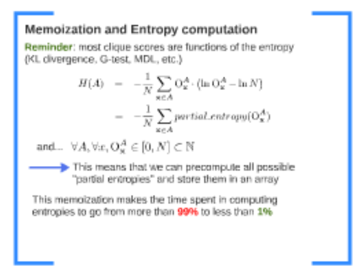


Memoization and Entropy computation

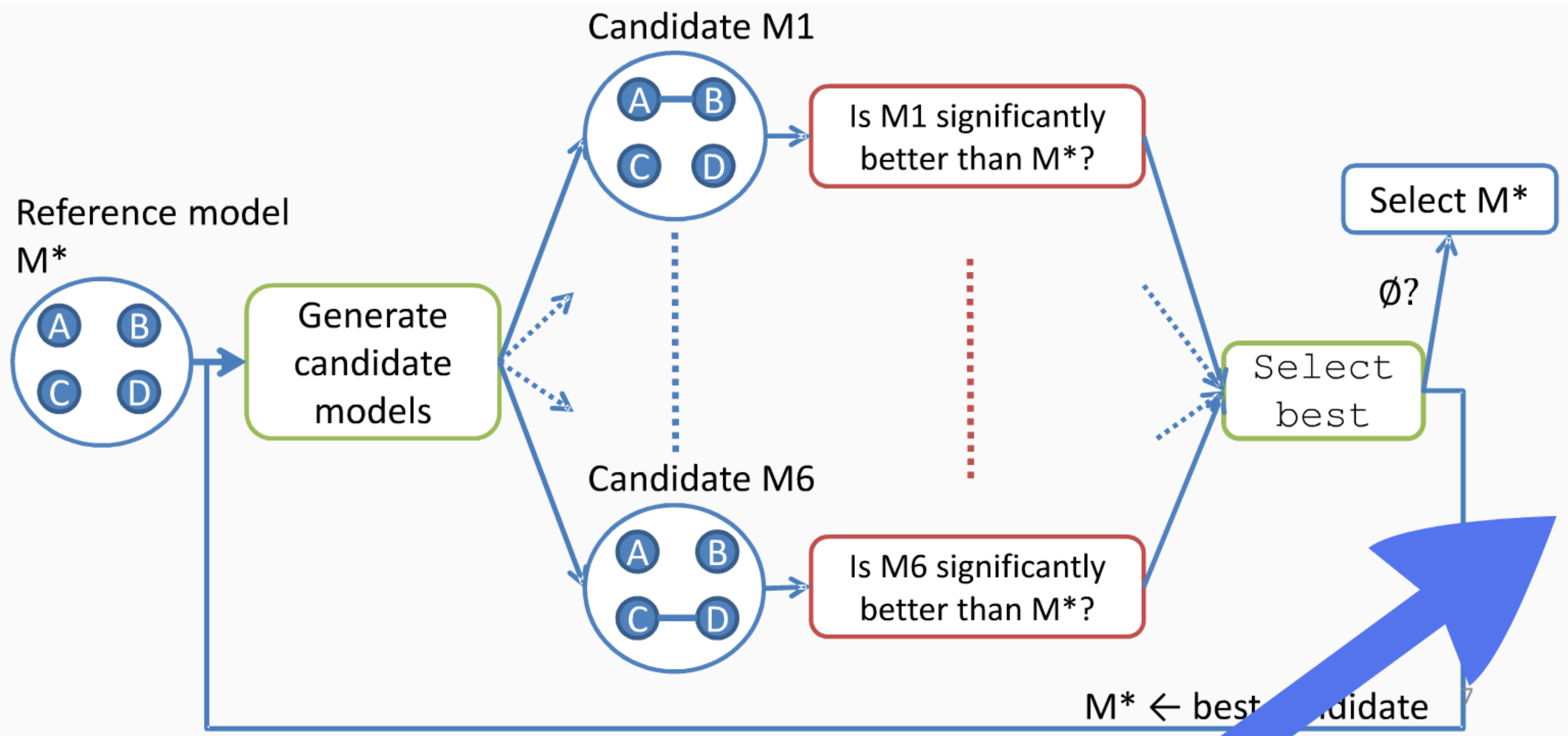
Reminder: most clique scores are functions of the entropy (KL divergence, G-test, MDL, etc.)

$$H(A) = -\frac{1}{N} \sum_{x \in A} O_x^A \cdot (\ln O_x^A - \ln N)$$
$$= -\frac{1}{N} \sum_{x \in A} partial\_entropy(O_x^A)$$

and... $\forall A, \forall x, O_x^A \in [0, N] \subset \mathbb{N}$

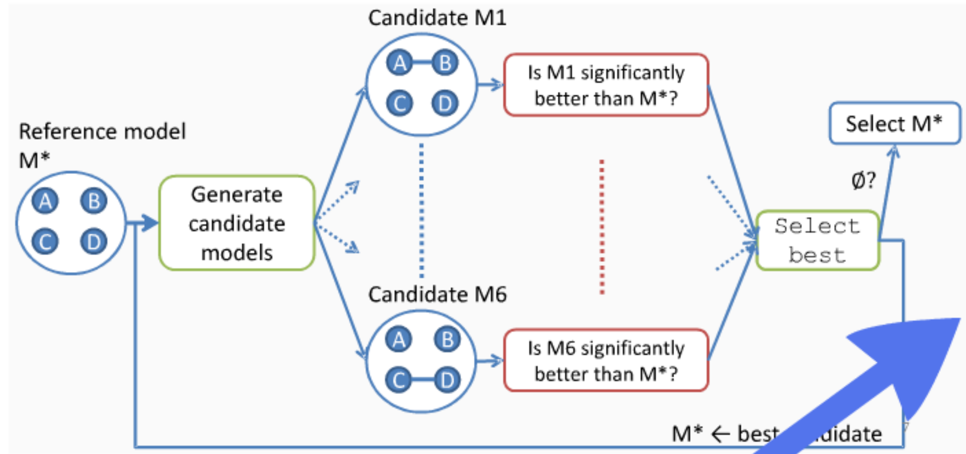
→ This means that we can precompute all possible "partial entropies" and store them in an array

This memoization makes the time spent in computing entropies to go from more than 99% to less than 1%



dered several times

Memoization



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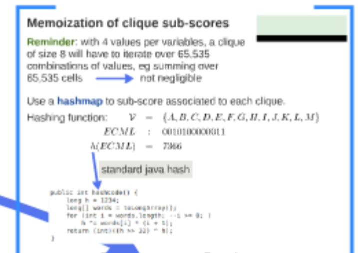

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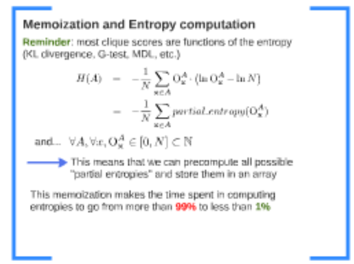

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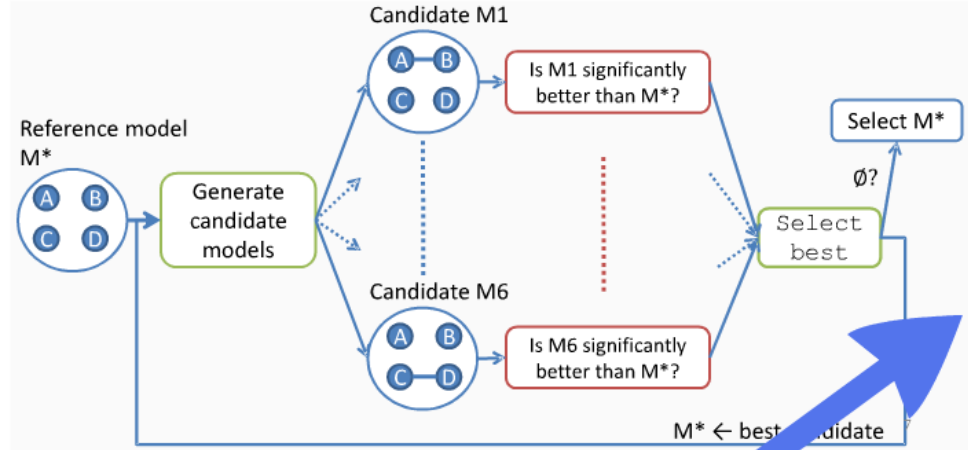

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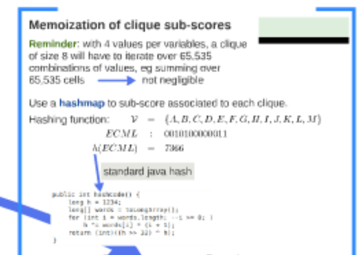

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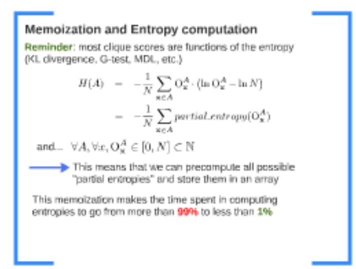

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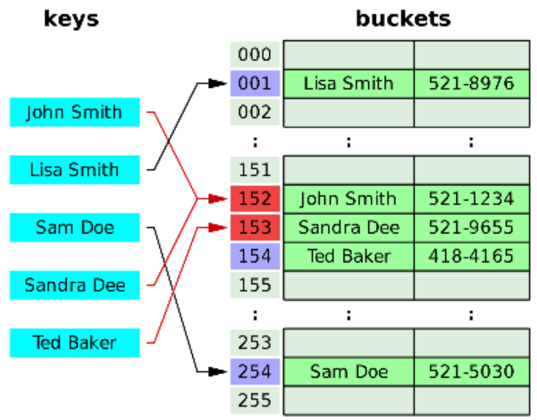

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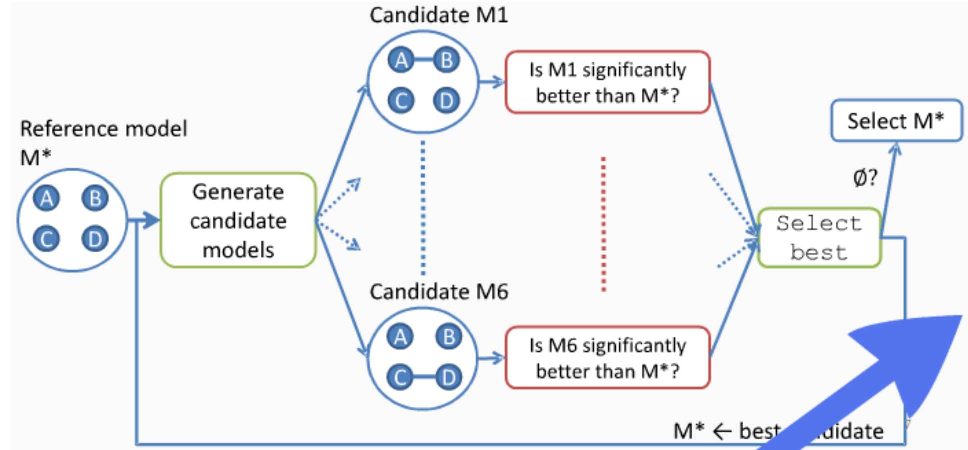

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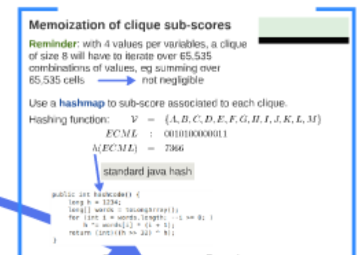

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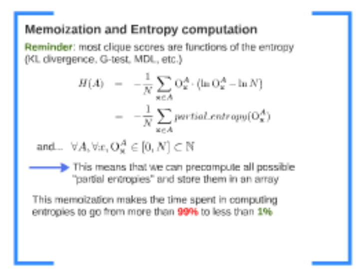
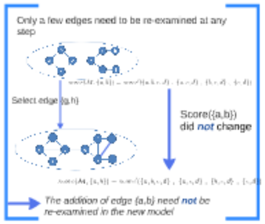


Addition of the same edge to different reference models

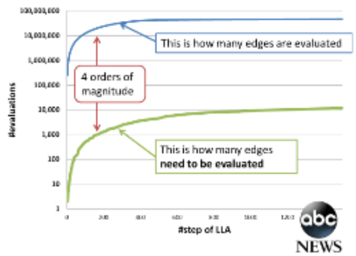
What we have seen so far:

- Evaluating the addition of an edge only depends upon 4 cliques of the graph

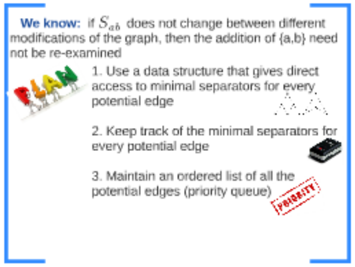
Our intuition

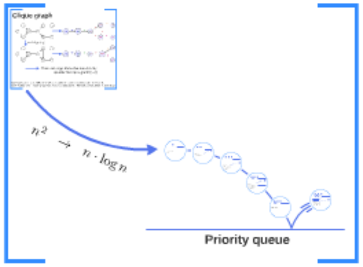


How often does that happen?

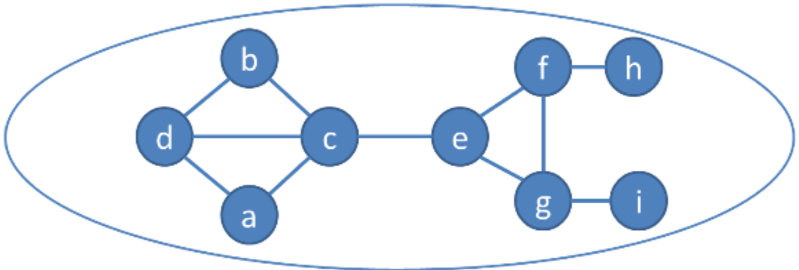


How can we use this information?





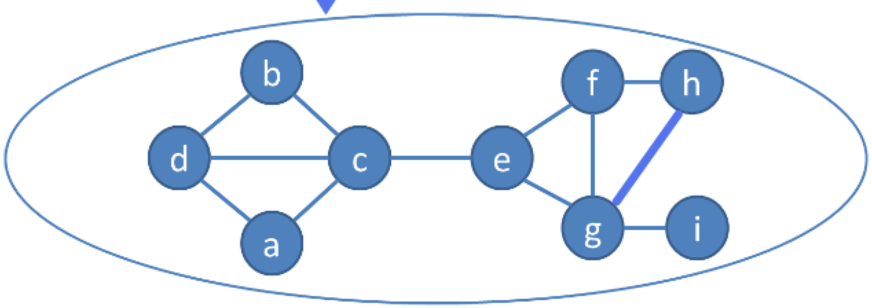
Only a few edges need to be re-examined at any step



$$score(\mathcal{M}, \{a,b\}) = score'(\{a,b,c,d\}\ ,\ \{a,c,d\}\ ,\ \{b,c,d\}\ ,\ \{c,d\})$$

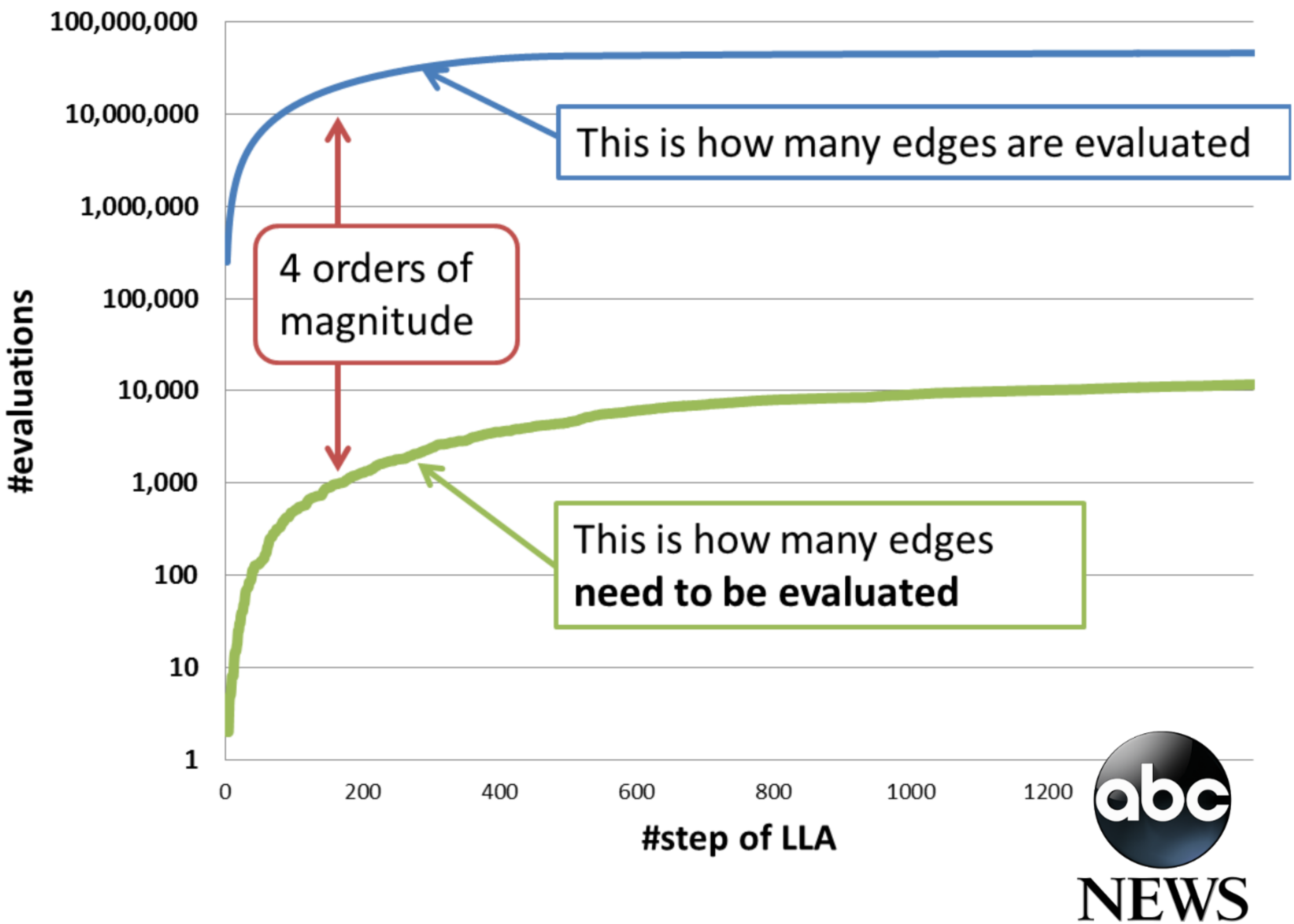
Select edge {g,h}

Score({a,b}) did ***not*** change



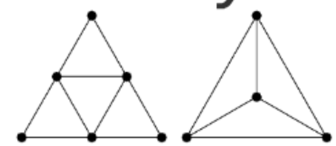
$$score(\mathcal{M}, \{a,b\}) = score'(\{a,b,c,d\}\ ,\ \{a,c,d\}\ ,\ \{b,c,d\}\ ,\ \{c,d\})$$

The addition of edge {a,b} need ***not*** *be re-examined in the new model*


100,000,000
10,000,000
1,000,000
100,000
10,000
1,000
100
10
1
#evaluations
0
200
400
600
800
1000
1200
#step of LLA
This is how many edges are evaluated
4 orders of magnitude
This is how many edges need to be evaluated
abc NEWS

We know: if S_{ab} does not change between different modifications of the graph, then the addition of {a,b} need not be re-examined

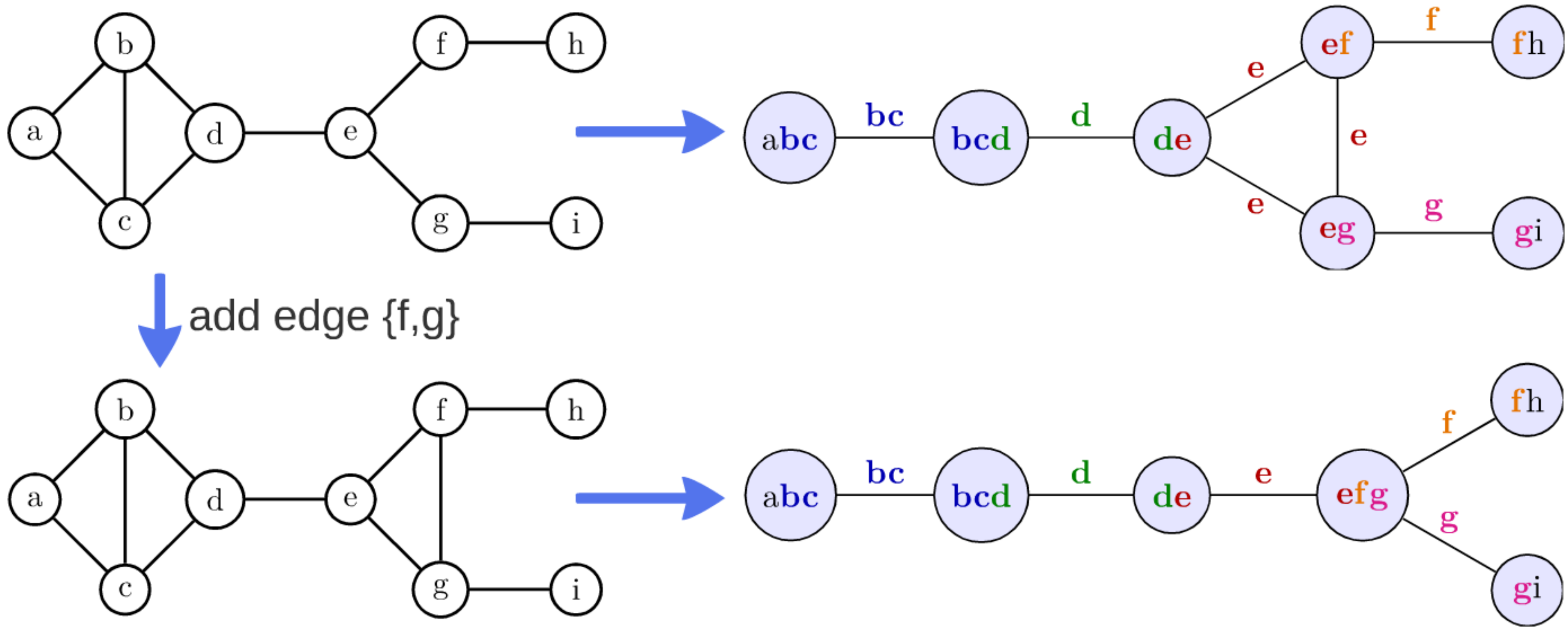
1. Use a data structure that gives direct access to minimal separators for every potential edge



2. Keep track of the minimal separators for every potential edge

3. Maintain an ordered list of all the potential edges (priority queue)

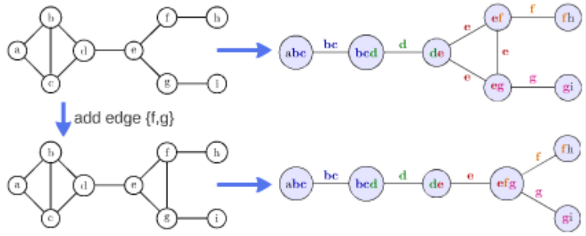
Clique graph



There are algorithms that can directly update the clique-graph [1,2]

[1] A. Deshpande *et al.*, "Efficient stepwise selection in decomposable models," in *UAI 2001*.
[2] F. Petitjean *et al.*, "Scaling log-linear analysis to datasets with thousands of variablesi" In *SDM 2015*.

Clique graph



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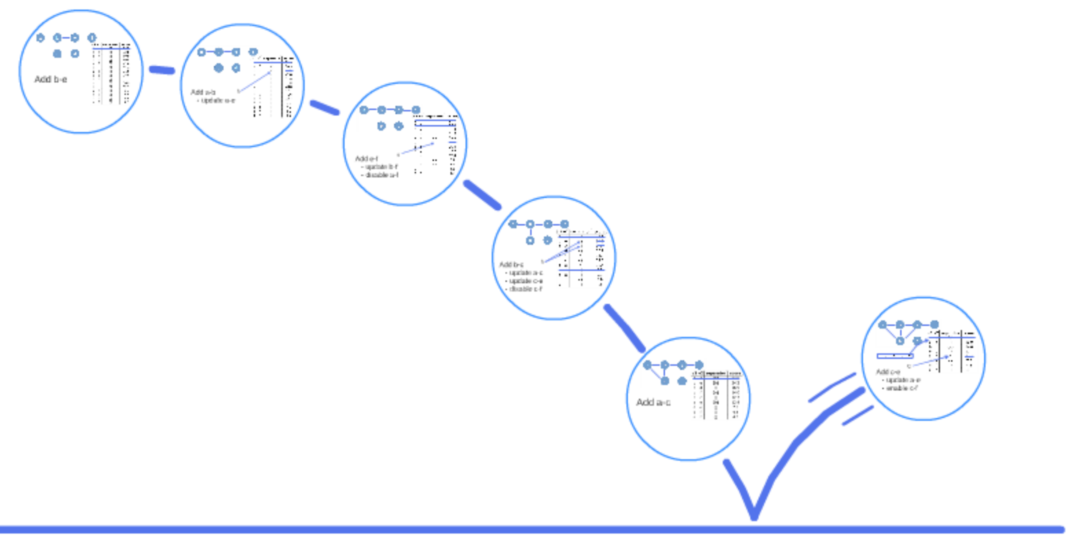
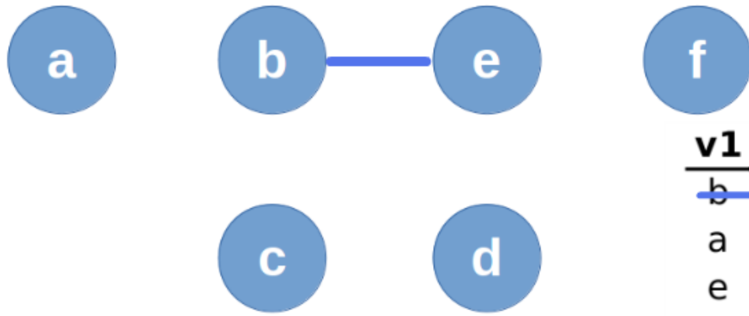
$$n^2 \rightarrow n \cdot \log n$$

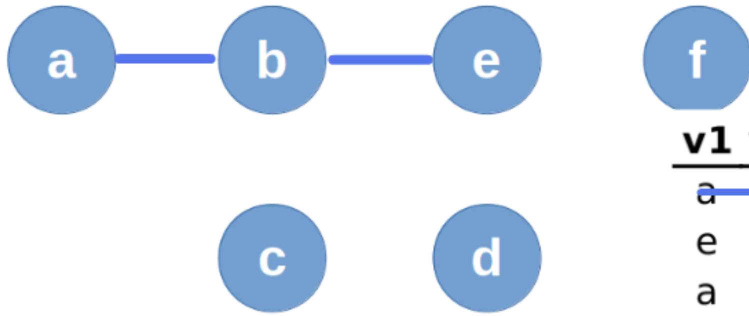


Priority queue



v1	v2	separator	score
~~b~~	~~e~~	~~{}~~	~~96.2~~
a	b	{}	72.8
e	f	{}	60.9
a	e	{}	49.5
a	f	{}	42.8
b	c	{}	31.4
c	e	{}	31.0
a	c	{}	28.8
b	f	{}	17.1
c	d	{}	16.9
b	c	{}	12.7
c	f	{}	8.1
d	e	{}	7.3
d	f	{}	4.8
e	f	{}	4.6

Add b-e

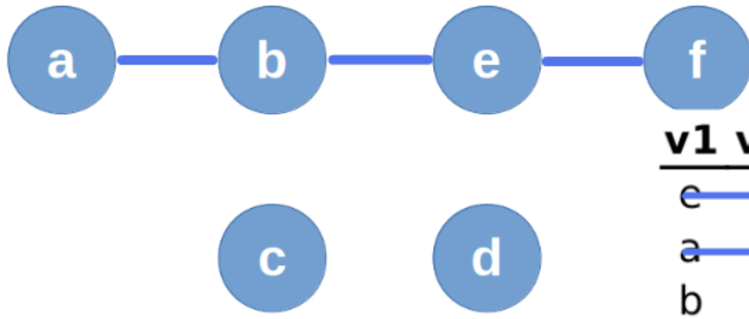


v1	v2	separator	score
~~a~~	~~b~~	~~{}~~	~~72.8~~
e	f	{}	60.9
a	e	{}	~~49.5~~
a	f	{}	42.8
b	e	{}	31.4
c	e	{}	31.0
a	c	{}	28.8
b	f	{}	17.1
c	d	{}	16.9
b	c	{}	12.7
c	f	{}	8.1
d	e	{}	7.3
d	f	{}	4.8
e	f	{}	4.6

b

Add a-b

- update a-e

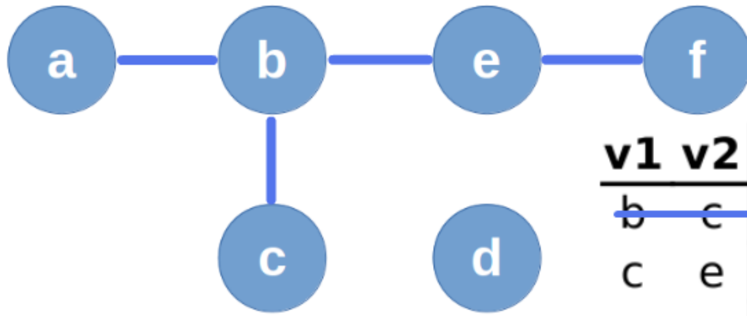


e

v1	v2	separator	score
~~e~~	~~f~~	~~{}~~	~~60.9~~
~~a~~	~~f~~	~~{}~~	~~42.8~~
b	c	{}	31.4
c	e	{}	31.0
a	c	{}	28.8
b	f	{}	~~17.1~~
c	d	{}	16.9
b	c	{}	12.7
a	e	{b}	12.4
c	f	{}	8.1
d	e	{}	7.3
d	f	{}	4.8
e	f	{}	4.6

Add e-f

- update b-f
- disable a-f

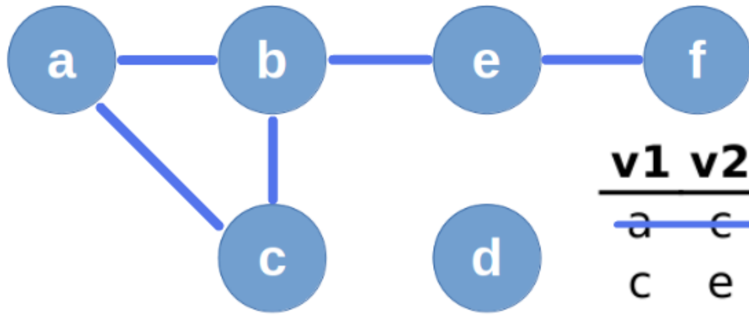


v1	v2	separator	score
~~b~~	~~c~~	~~{}~~	~~31.4~~
c	e	{}	~~31.0~~
a	c	{}	~~28.8~~
c	d	{}	16.9
b	f	{e}	14.0
b	c	{}	12.7
a	e	{b}	12.4
~~c~~	~~f~~	~~{}~~	~~8.1~~
d	e	{}	7.3
d	f	{}	4.8
e	f	{}	4.6

b

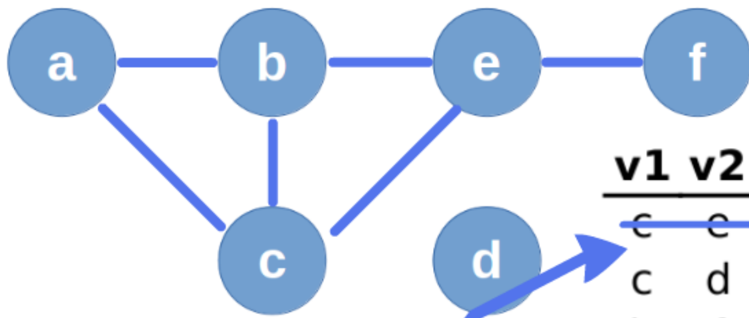
Add b-c

- update a-c
- update c-e
- disable c-f



v1	v2	separator	score
~~a~~	~~c~~	~~{b}~~	~~84.5~~
c	e	{b}	24.2
c	d	{}	16.9
b	f	{e}	14.0
b	c	{}	12.7
a	e	{b}	12.4
d	e	{}	7.3
d	f	{}	4.8
e	f	{}	4.6

Add a-c



v1	v2	separator	score
~~c~~	~~e~~	~~{b}~~	~~24.2~~
c	d	{}	16.9
b	f	{e}	14.0
b	c	{}	12.7
a	e	{b}	~~12.4~~
d	e	{}	7.3
d	f	{}	4.8
e	f	{}	4.6

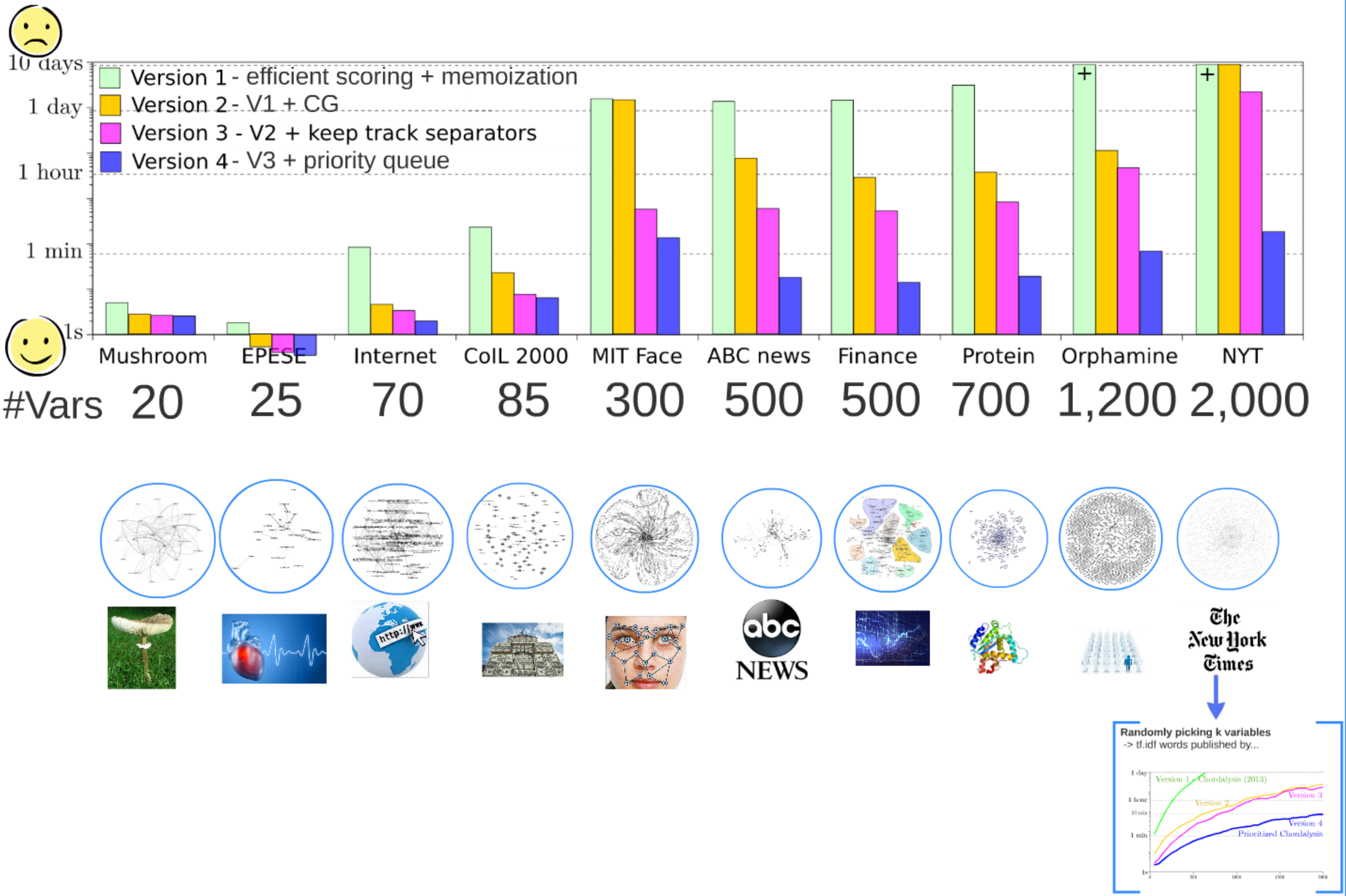
c	f	{e}	49.4

c

Add c-e

- update a-e
- enable c-f

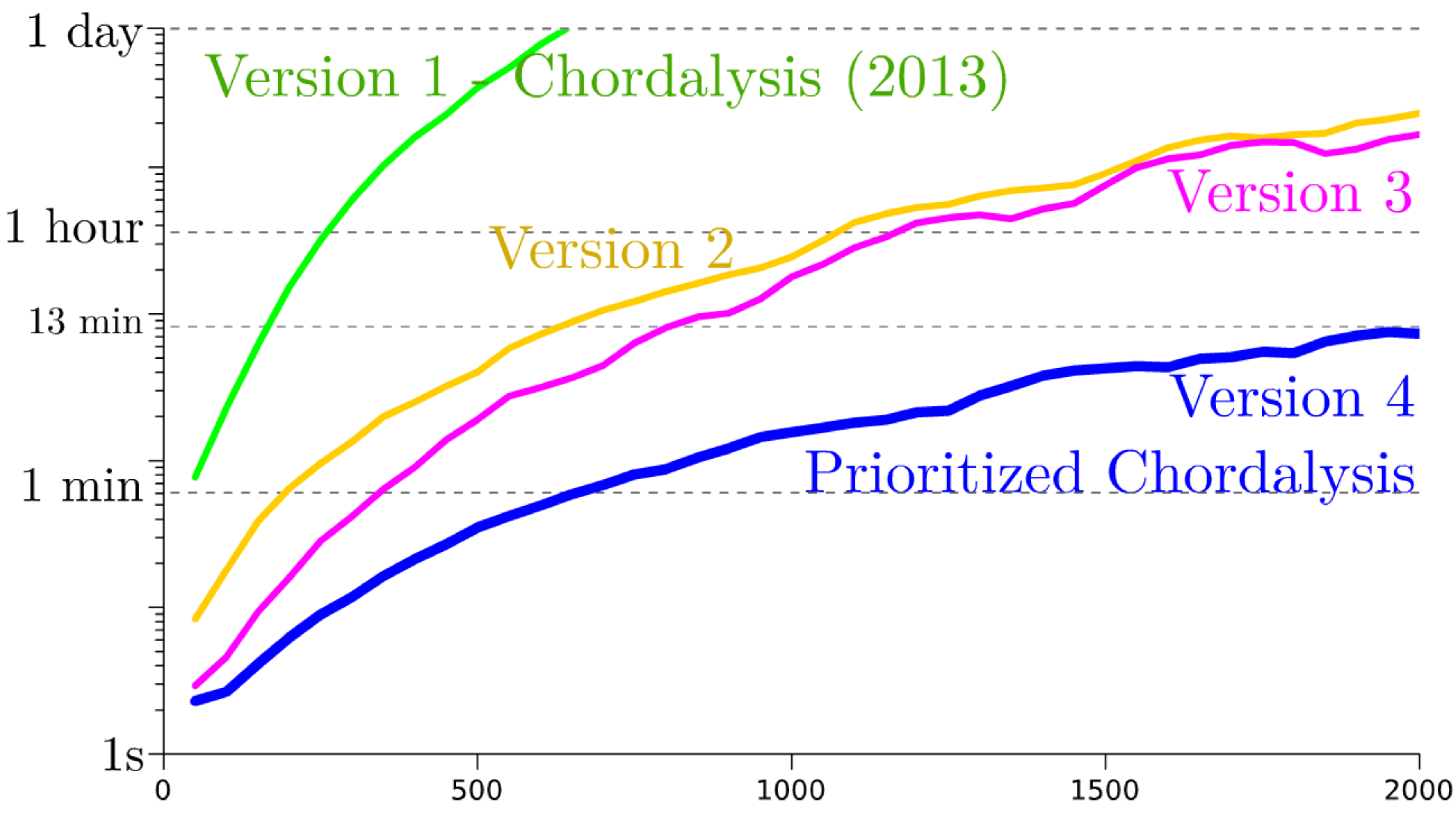
How fast can we get?



Randomly picking k variables

-> tf.idf words published by...

The New York Times



Scalable learning of graphical models

Introduction - Motivation

Graphical models 101

Graph theory

Evaluation - Scoring

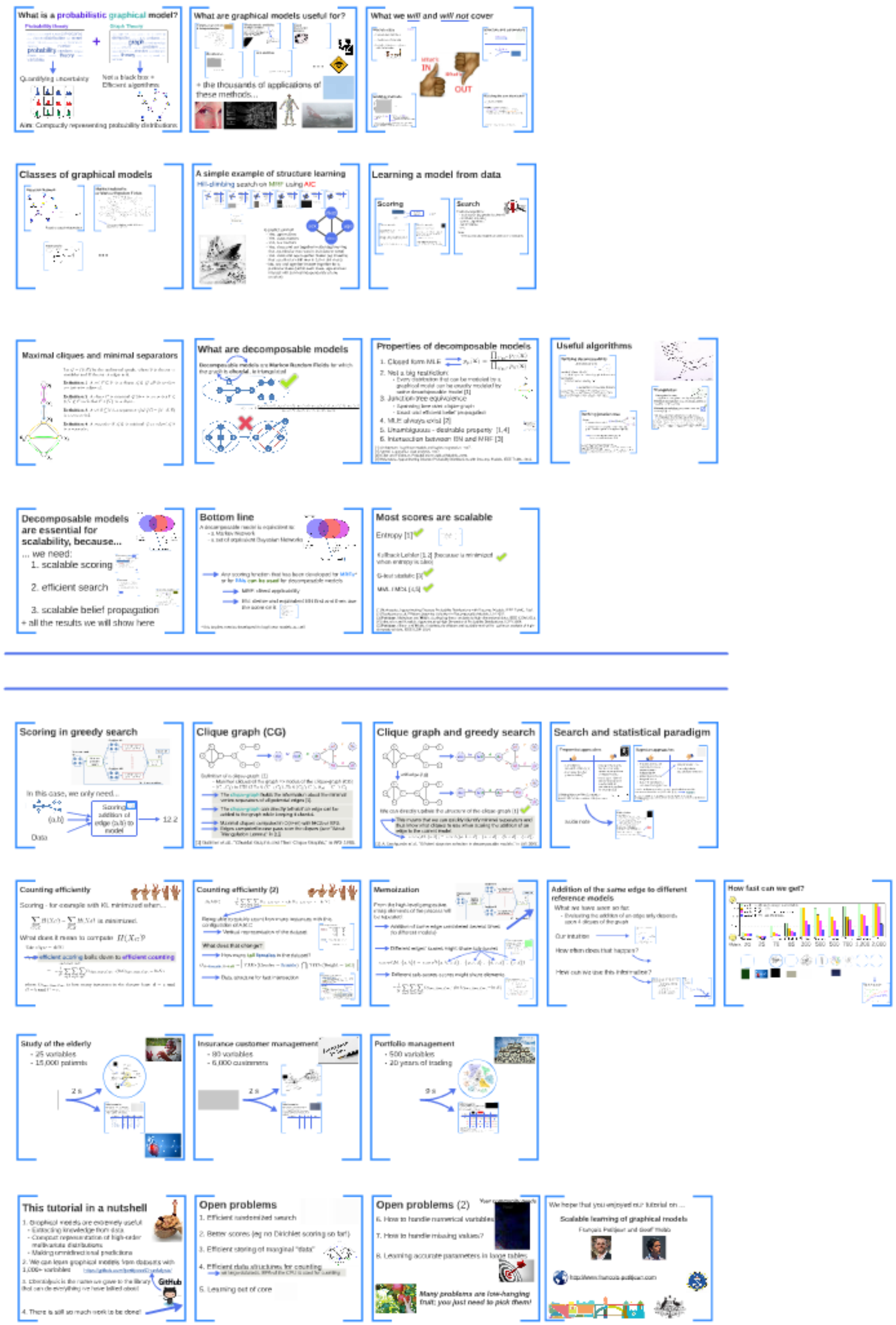
Break

Efficient search

The nitty-gritty

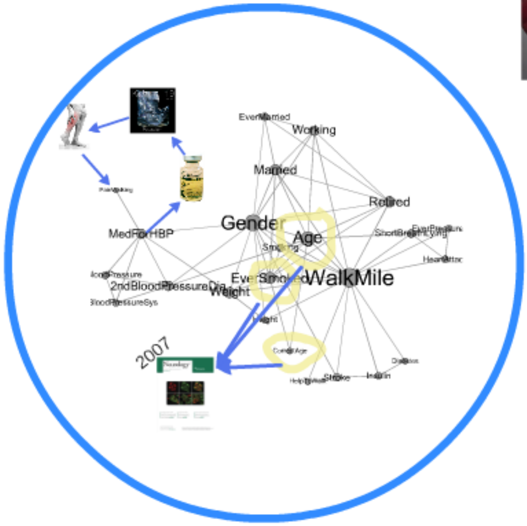
Use cases

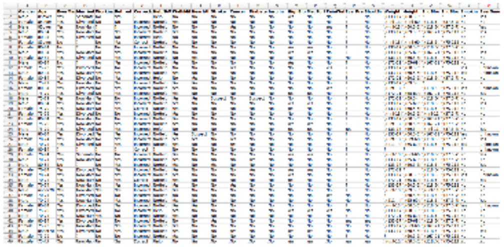
Wrapping up!



Study of the elderly

- 25 variables
- 15,000 patients





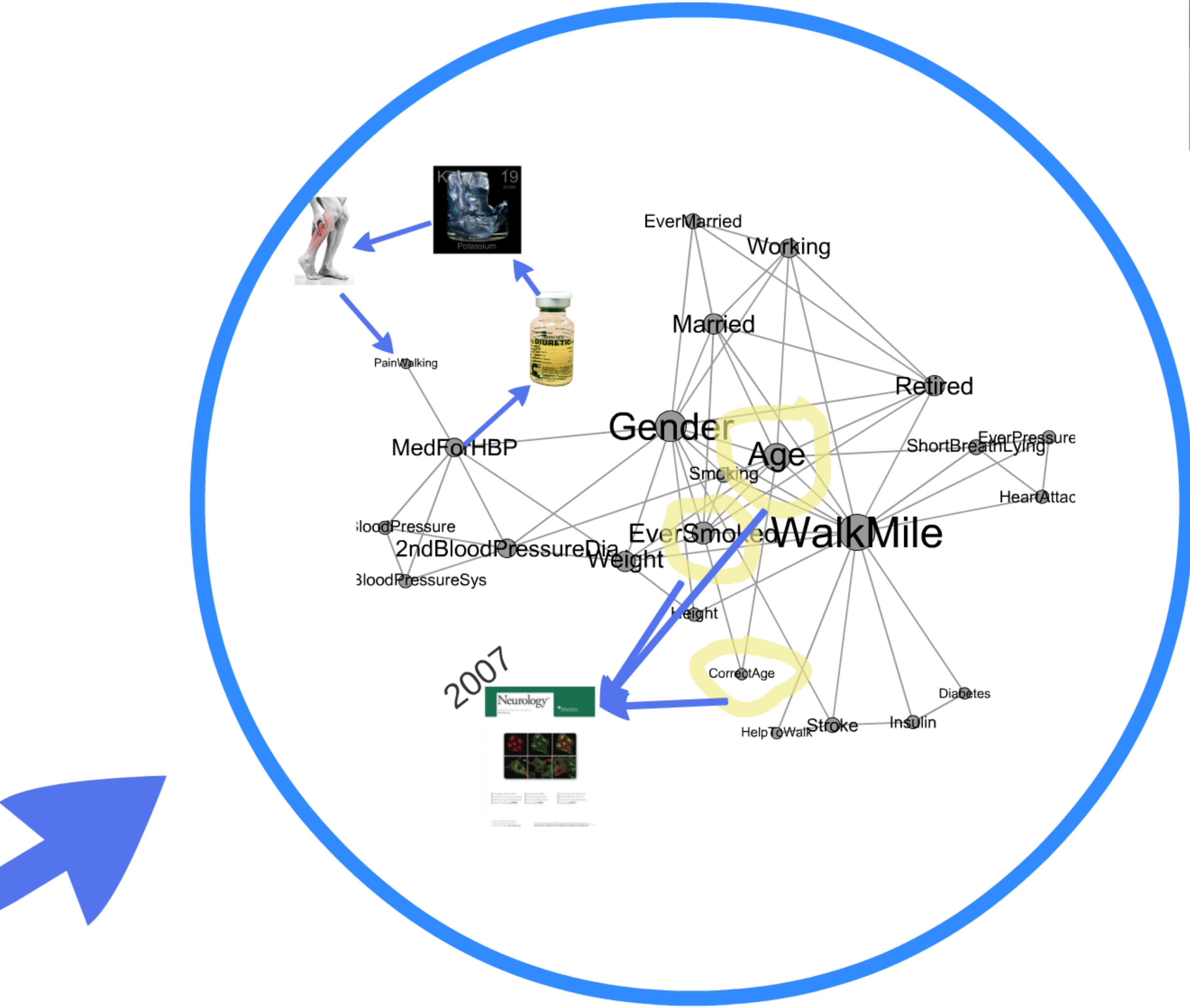
2 s

Belief propagation

New patient, Lan, is visiting her new GP; the GP wants to check her risk of getting a few diseases: stroke, diabetes, heart attack.

evidence	stroke	diabetes	heart attack
female under 70	*5%*	*15%*	*10%*
+ married	*5%*	*15%*	*9%*
+ smoking	*7%*	*17%*	*12%*
+ BP=17/10	*8%*	*17%*	*13%*
+ no help to walk	*5%*	*16%*	*12%*
+ quit smoking?	*4%*	*14%*	*9%*

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y
1	**Gender**	**Age**	**EverMai**	**Married**	**Workin**	**Retired**	**Correct**	**HelpToV**	**WalkMil**	**HeartAt**	**Stroke**	**Cancer**	**Diabete**	**Insulin**	**HighBlo**	**MedForI**	**PainWal**	**EverPre**	**ShortBr**	**Weight**	**Height**	**2ndBloo**	**2ndBloo**	**Smokin**	**EverSmo**
2	Male	85over	Yes	Separate	No	Yes	?	Help	No	No	No	No	No	No	Yes	No	No	?	No	\'(133-17	\'(60.5-65	?	?	No	Yes
3	Female	85over	Yes	Divorced	No	No	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	?	No	\'(-inf-133	\'(-inf-60.	\'(118.5-1	\'(37.5-75	No	No
4	Male	85over	Yes	NowMarr	?	?	Incorrect	NoHelp	No	No	No	Yes	No	No	No	No	Yes	?	No	\'(-inf-133	\'(60.5-65	\'(118.5-1	\'(37.5-75	No	Yes
5	Male	80-84	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	?	No	\'(133-17	\'(69.5-in	\'(167-21	\'(75-112	No	Yes
6	Female	80-84	Yes	Divorced	No	No	Incorrect	Help	No	No	No	No	No	No	No	No	No	?	No	?	?	?	?	No	No
7	Female	85over	No	?	No	Yes	Correct	NoHelp	Yes	No	No	No	No	No	No	No	No	?	No	\'(-inf-133	\'(-inf-60.	\'(118.5-1	\'(75-112	No	No
8	Female	80-84	No	?	No	No	Incorrect	NoHelp	No	No	No	No	No	No	Yes	Yes	No	?	No	\'(133-17	\'(60.5-65	\'(118.5-1	\'(37.5-75	No	No
9	Male	80-84	Yes	NowMarr	No	Yes	Incorrect	NoHelp	No	No	No	No	No	No	Yes	Yes	No	?	No	\'(133-17	\'(65-69.5	\'(167-21	\'(75-112	No	Yes
10	Female	80-84	Yes	Divorced	No	Yes	Incorrect	NoHelp	No	Yes	No	No	No	No	No	No	No	No	No	\'(133-17	\'(60.5-65	\'(118.5-1	\'(75-112	No	No
11	Male	80-84	No	?	No	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	Yes	No	No	?	No	\'(172-21	\'(65-69.5	\'(167-21	\'(75-112	No	No
12	Female	75-79	Yes	Divorced	No	No	Incorrect	NoHelp	No	No	No	Yes	No	No	No	No	Yes	?	No	\'(133-17	\'(60.5-65	?	?	Yes	Unknown
13	Male	80-84	Yes	NowMarr	Yes	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	?	No	\'(133-17	\'(60.5-65	\'(118.5-1	\'(75-112	Yes	Unknown
14	Female	80-84	Yes	Divorced	No	Yes	Incorrect	Help	No	Yes	No	No	No	No	No	No	No	?	No	\'(172-21	\'(60.5-65	\'(118.5-1	\'(37.5-75	No	No
15	Male	75-79	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	Yes	No	No	No	No	No	No	No	?	No	\'(133-17	\'(69.5-in	?	?	No	No
16	Female	80-84	Yes	Divorced	No	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	Yes	?	No	\'(-inf-133	?	\'(118.5-1	\'(75-112	Yes	Unknown
17	Male	75-79	Yes	Divorced	No	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	Yes	Yes	No	?	No	\'(172-21	\'(65-69.5	\'(167-21	\'(75-112	No	Yes
18	Male	75-79	Yes	NowMarr	Yes	No	Incorrect	NoHelp	Yes	Yes	Suspect	No	Suspect	No	No	No	No	?	No	\'(172-21	\'(69.5-in	\'(118.5-1	\'(37.5-75	No	No
19	Male	80-84	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	Yes	Yes	No	?	No	\'(133-17	\'(65-69.5	\'(118.5-1	\'(75-112	No	Yes
20	Female	75-79	Yes	NowMarr	No	Yes	Correct	NoHelp	Yes	No	No	No	No	No	Yes	No	No	?	No	\'(133-17	\'(60.5-65	\'(118.5-1	\'(75-112	No	No
21	Female	75-79	Yes	Divorced	No	No	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	?	No	\'(-inf-133	\'(60.5-65	\'(-inf-118	\'(37.5-75	No	No
22	Female	80-84	Yes	NowMarr	No	No	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	?	No	\'(133-17	\'(60.5-65	\'(167-21	\'(37.5-75	No	No
23	Female	75-79	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	Yes	Yes	No	?	No	\'(-inf-133	\'(60.5-65	\'(167-21	\'(37.5-75	No	No
24	Male	75-79	Yes	Divorced	No	Yes	Incorrect	Help	No	No	No	No	No	No	Yes	Yes	No	Yes	No	\'(133-17	\'(69.5-in	\'(-inf-118	\'(37.5-75	No	Yes
25	Male	80-84	Yes	Divorced	Yes	Yes	Incorrect	NoHelp	Yes	Suspect	No	No	No	No	No	No	No	?	No	\'(133-17	\'(60.5-65	\'(167-21	\'(75-112	Yes	Unknown
26	Female	70-74	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	Yes	No	Yes	No	No	No	No	No	?	No	\'(-inf-133	\'(-inf-60.	\'(118.5-1	\'(37.5-75	No	Yes
27	Male	70-74	Yes	Divorced	No	Yes	Incorrect	NoHelp	Yes	No	No	Yes	No	No	No	No	No	?	No	\'(211-inf	\'(65-69.5	\'(118.5-1	\'(75-112	Yes	Unknown
28	Female	80-84	?	?	?	?	Correct	?	No	No	No	No	No	No	No	No	No	?	No	?	?	?	?	?	Unknown
29	Female	70-74	Yes	Separate	No	No	Incorrect	NoHelp	Yes	No	No	No	No	No	Yes	Yes	No	?	No	\'(-inf-133	\'(-inf-60.	\'(118.5-1	\'(37.5-75	No	Yes
30	Male	70-74	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	No	No	No	Yes	No	No	No	No	?	No	\'(133-17	\'(65-69.5	\'(118.5-1	\'(37.5-75	No	No
31	Male	80-84	No	?	No	Yes	Incorrect	NoHelp	Yes	No	Yes	No	No	No	No	No	No	?	No	\'(133-17	\'(60.5-65	\'(118.5-1	\'(75-112	No	Yes
32	Female	70-74	Yes	Divorced	Yes	Yes	Incorrect	NoHelp	Yes	No	No	No	Yes	No	Yes	Yes	No	?	No	\'(172-21	?	\'(118.5-1	\'(75-112	No	No
33	Female	70-74	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	Yes	No	No	?	No	\'(133-17	\'(60.5-65	\'(118.5-1	\'(75-112	Yes	Unknown
34	Male	70-74	Yes	NowMarr	No	Yes	Correct	NoHelp	Yes	No	No	No	No	No	Yes	No	No	?	No	\'(172-21	\'(65-69.5	\'(118.5-1	\'(37.5-75	No	Yes
35	Female	70-74	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	No	No	Yes	No	No	No	No	No	?	No	\'(133-17	\'(60.5-65	\'(118.5-1	\'(75-112	No	No
36	Male	under70	Yes	NowMarr	Yes	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	?	No	\'(-inf-133	\'(69.5-in	\'(-inf-118	\'(37.5-75	No	No
37	Female	70-74	Yes	Divorced	No	No	Incorrect	NoHelp	No	No	No	No	No	No	No	No	Yes	No	No	?	\'(60.5-65	\'(118.5-1	\'(37.5-75	No	No
38	Male	70-74	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	No	No	\'(211-inf	\'(65-69.5	\'(118.5-1	\'(37.5-75	No	Yes
39	Female	80-84	Yes	Divorced	No	Yes	Incorrect	NoHelp	Yes	Yes	No	No	No	No	No	No	No	?	No	\'(-inf-133	\'(60.5-65	\'(118.5-1	\'(37.5-75	Yes	Unknown
40	Female	75-79	Yes	Divorced	No	No	Incorrect	Help	Yes	No	No	No	No	No	Yes	Yes	Yes	No	No	\'(133-17	\'(60.5-65	\'(118.5-1	\'(75-112	No	No
41	Male	70-74	Yes	NowMarr	Yes	No	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	?	No	\'(172-21	\'(65-69.5	\'(118.5-1	\'(75-112	No	No
42	Female	80-84	Yes	Divorced	No	No	Incorrect	NoHelp	No	No	Yes	No	No	No	No	No	No	Yes	Yes	\'(133-17	\'(65-69.5	\'(118.5-1	\'(37.5-75	No	No
43	Female	75-79	Yes	NowMarr	No	Yes	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	No	No	\'(133-17	\'(-inf-60.	\'(167-21	\'(75-112	No	No
44	Male	under70	Yes	Divorced	No	Yes	Incorrect	NoHelp	Yes	No	No	Yes	No	No	Yes	Yes	No	No	No	\'(133-17	\'(60.5-65	\'(118.5-1	\'(75-112	No	Yes
45	Female	75-79	No	?	No	No	Incorrect	NoHelp	Yes	No	No	No	No	No	No	No	No	?	No	?	\'(60.5-65	\'(167-21	\'(75-112	No	No
46	Female	75-79	Yes	NowMarr	No	Yes	Correct	Help	No	Yes	No	No	No	No	Yes	Yes	No	Yes	Yes	\'(-inf-133	\'(60.5-65	\'(118.5-1	\'(75-112	No	No


EverMarried
Working
Married
Retired
Gender
Age
Smoking
EverSmoked
WalkMile
ShortBreathLying
EverPressure
PainWalking
MedForHBP
2ndBloodPressureDia
Weight
Height
CorrectAge
Diabetes
Insulin
Stroke
HelpToWalk
2007
Neurology
DIURETIC
Potassium
19

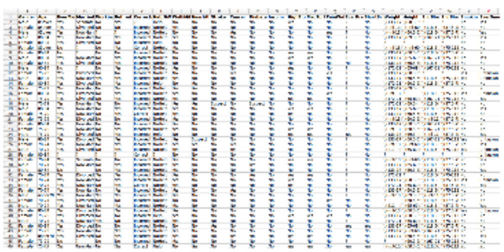
Belief propagation

New patient, Lan, *is visiting her new GP; the GP wants to check her risk of getting a few diseases: stroke, diabetes, heart attack.*

evidence	stroke	diabetes	heart attack
female under 70	*5%*	*15%*	*10%*
+ married	*5%*	*15%*	*9%*
+ smoking	*7%*	*17%*	*12%*
+ BP=17/10	*8%*	*17%*	*13%*
+ no help to walk	*5%*	*16%*	*12%*
+ quit smoking?	*4%*	*14%*	*9%*

Study of the elderly

- 25 variables
- 15,000 patients



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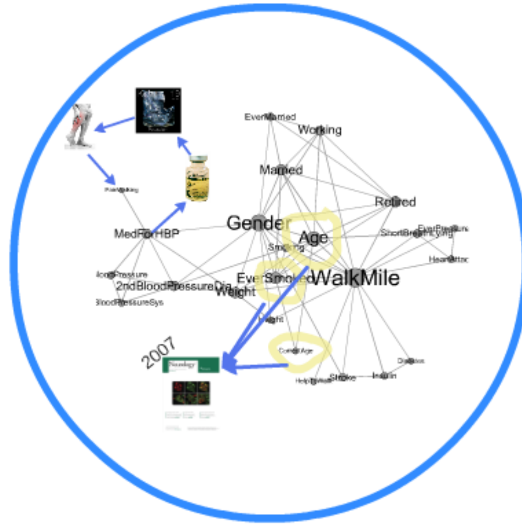
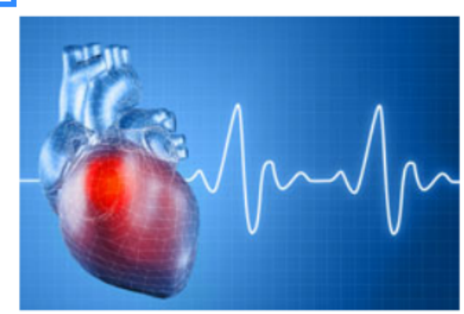


Belief propagation

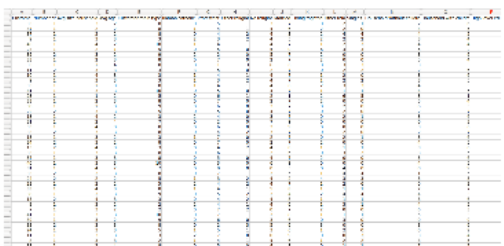
New patient, Lan, is visiting her new GP; the GP wants to check her risk of getting a few diseases: stroke, diabetes, heart attack.

evidence	stroke	diabetes	heart attack
female under 70	*5%*	*15%*	*10%*
+ married	*5%*	*15%*	*9%*
+ smoking	*7%*	*17%*	*12%*
+ BP=17/10	*8%*	*17%*	*13%*
+ no help to walk	*5%*	*16%*	*12%*
+ quit smoking?	*4%*	*14%*	*9%*



Insurance customer management

- 80 variables
- 6,000 customers



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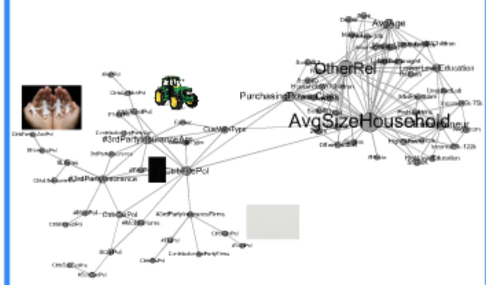
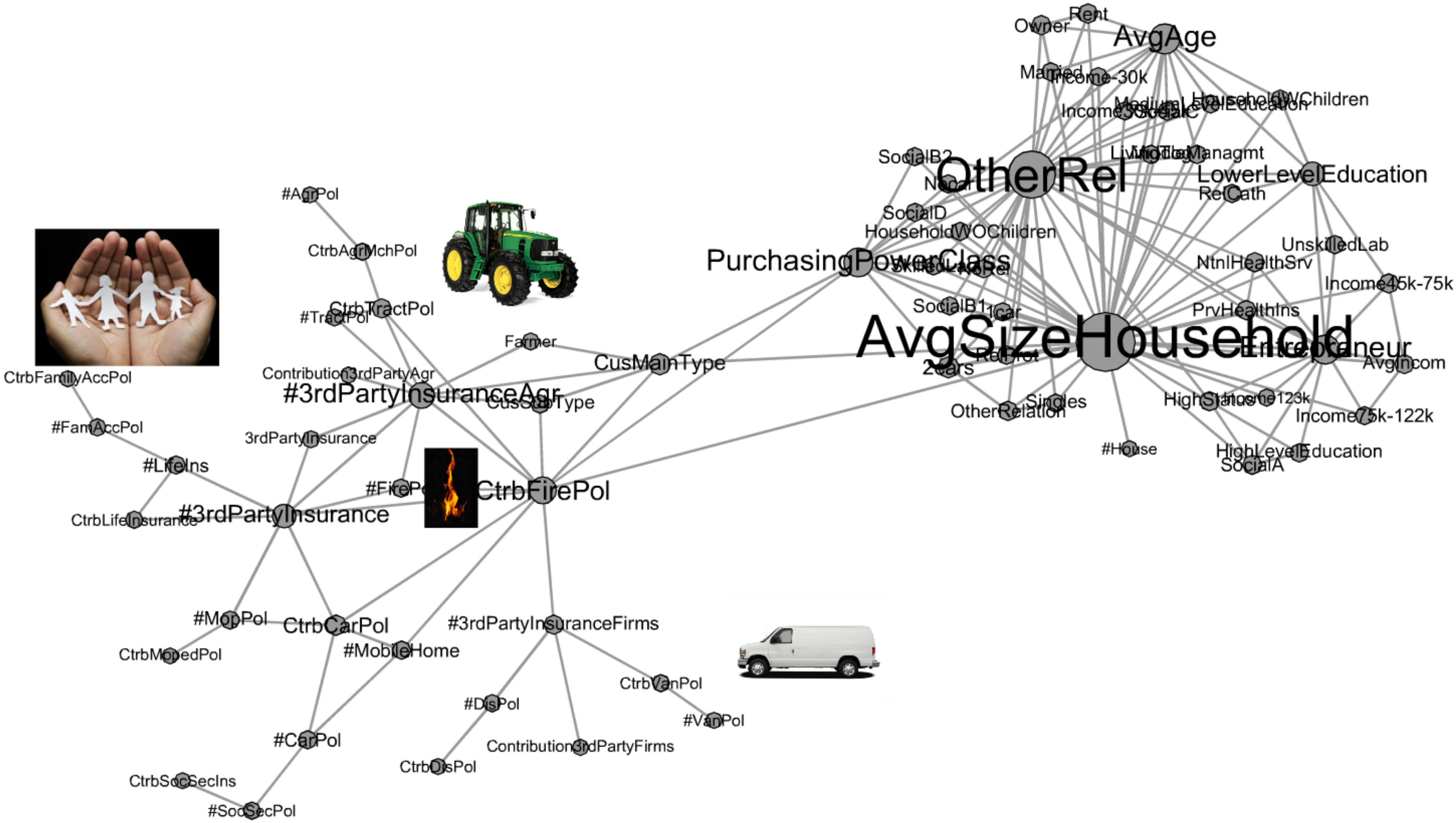


Belief propagation

New customer, Mat, is visiting his new branch; the customer representative takes the opportunity to check potential for new insurance policies.

evidence	fire	van	life

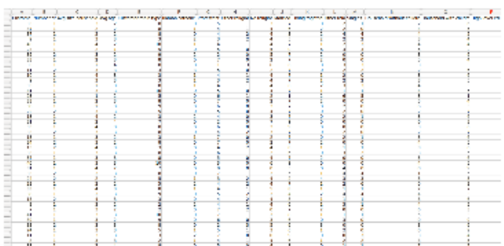
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Customer	Number of	Avg size household	Avg age	Customer main type	Roman catholic	Protestant	Other religior	No religior	Married	Living togethe	Other rela	Singles	Household without childre	Household with childre	High level edu
33	1	3	2	8	0	5	1	3	7	0	2	1	2	6	
37	1	2	2	8	1	4	1	4	6	2	2	0	4	5	
37	1	2	2	8	0	4	2	4	3	2	4	4	4	2	
9	1	3	3	3	2	3	2	4	5	2	2	2	3	4	
40	1	4	2	10	1	4	1	4	7	1	2	2	4	4	
23	1	2	1	5	0	5	0	5	0	6	3	3	5	2	
39	2	3	2	9	2	2	0	5	7	2	0	0	3	6	
33	1	2	3	8	0	7	0	2	7	2	0	0	5	4	
33	1	2	4	8	0	1	3	6	6	0	3	3	3	3	
11	2	3	3	3	3	5	0	2	7	0	2	2	2	6	
10	1	4	3	3	1	4	1	4	7	1	2	0	3	6	
9	1	3	3	3	1	3	2	4	7	1	2	2	3	5	
33	1	2	3	8	1	4	1	4	6	2	3	3	4	3	
41	1	3	3	10	0	5	0	4	7	1	1	1	4	5	
23	1	1	2	5	0	6	1	2	1	2	6	5	3	1	
33	1	2	3	8	0	7	0	2	7	2	0	0	5	4	
38	1	2	3	9	0	6	0	3	7	0	2	0	6	3	
22	2	3	3	5	0	5	0	4	7	0	2	0	2	7	
13	1	4	2	3	2	4	0	3	7	0	2	1	3	6	
31	1	2	4	7	0	2	0	7	9	0	0	0	6	3	
33	1	4	3	8	0	6	0	3	9	0	0	0	3	6	
33	2	3	3	8	0	4	2	3	7	0	2	0	2	7	
13	1	3	2	3	1	7	0	2	7	0	2	1	3	6	
34	2	3	2	8	0	7	0	2	7	2	0	0	4	5	
13	2	4	3	3	0	4	2	4	8	1	1	1	3	6	
33	1	3	3	8	0	6	1	2	6	0	3	2	3	5	
37	1	3	3	8	0	5	0	4	7	2	0	0	3	6	
40	1	3	3	10	0	3	0	6	9	0	0	0	4	5	
31	1	4	2	7	0	9	0	0	5	0	4	0	0	9	
33	2	2	3	8	0	7	1	2	5	1	4	4	1	5	
24	2	2	2	5	1	3	2	4	2	4	3	3	3	3	
23	1	2	2	5	2	4	2	4	7	1	2	6	0	3	
33	1	4	3	8	1	4	1	5	8	1	1	1	2	7	
38	1	2	3	9	0	5	2	2	4	2	4	4	4	3	
13	2	4	3	3	0	4	2	4	8	1	1	1	3	6	
8	1	3	2	2	2	4	1	4	6	1	3	1	4	5	
7	1	3	2	2	0	7	2	0	7	2	0	0	6	3	
41	1	3	3	10	0	7	1	2	8	1	1	1	5	3	
39	1	3	2	9	0	6	0	3	6	0	3	0	0	9	
33	2	3	3	8	0	2	3	5	6	3	0	0	3	6	
24	1	3	3	5	1	5	1	3	6	1	2	0	0	9	
11	1	3	3	3	2	7	0	0	9	0	0	2	3	4	
8	1	3	3	2	1	4	1	4	6	1	2	2	3	5	
33	1	2	4	8	0	5	0	4	8	0	1	1	7	2	



Owner
Rent
AvgAge
Married
Income-30k
HouseholdWChildren
MediumLevelEducation
SocialB2
OtherRel
LowerLevelEducation
RelCath
SocialD
HouseholdWOChildren
PurchasingPowerClass
UnskilledLab
NtnlHealthSrv
Income45k-75k
SocialB1
1car
PrvHealthIns
AvgSizeHousehold
Entrepreneur
AvgIncom
2cars
Singles
OtherRelation
HighStatus
Income123k
Income75k-122k
#House
HighLevelEducation
SocialA
#AgrPol
CtrbAgrMchPol
CtrbTractPol
#TractPol
Farmer
CusMainType
Contribution3rdPartyAgr
#3rdPartyInsuranceAgr
CusSubType
CtrbFamilyAccPol
#FamAccPol
3rdPartyInsurance
#LifeIns
#FirePol
CtrbFirePol
CtrbLifeInsurance
#3rdPartyInsurance
#MopPol
CtrbCarPol
#3rdPartyInsuranceFirms
CtrbMopedPol
#MobileHome
CtrbVanPol
#DisPol
#VanPol
#CarPol
Contribution3rdPartyFirms
CtrbDisPol
CtrbSocSecIns
#SocSecPol

Insurance customer management

- 80 variables
- 6,000 customers



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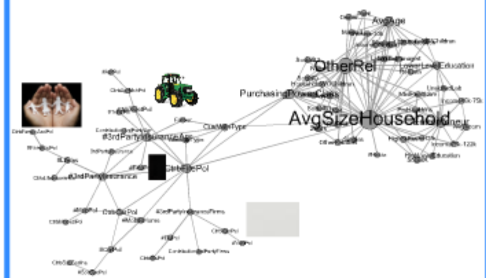


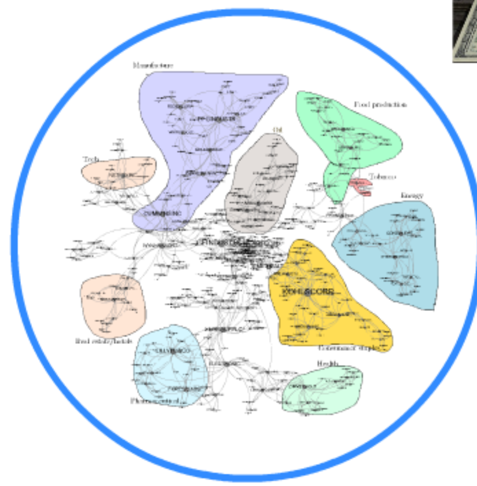
Belief propagation

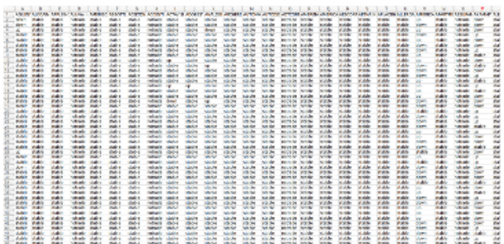
New customer, Mat, is visiting his new branch; the customer representative takes the opportunity to check potential for new insurance policies.

evidence	fire	van	life

Portfolio management

- 500 variables
- 20 years of trading





9 s

Belief propagation

Financial adviser wants to see how the market might behave given a few speculations over stocks.

evidence			NETFLIX		MERCK		RALPH LAUREN	
	↘	↗	↘	↗	↘	↗	↘	↗
prior	19%	19%	23%	22%	12%	13%	23%	25%
Seagate ↗	21%	21%	27%	30%	13%	13%	—	
Johnson-Johnson ↘	21%	20%	—		34%	15%	24%	25%
↘	26%	20%	34%	26%	—		23%	25%
a ↗	—		—		—		25%	37%

- Netflix needs drives?
- Merck and J&J are in the same cluster
- http://www.buyupside.com/ says AMAZN and RL have 0.89 correlation coefficient
    - External factor? Sales of Ralph Lauren on Amazon.com?

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	
1	AGILEN	ALCOAIN	APPLEINC	AbbVieIn	AMERISO	ABBOTTL	ACELTDN	ACCENTU	ACTAVISI	ADOBESY	ANALOGI	ARCHERD	AUTOMAT	AUTODES	ADTCORF	AMERENC	AMERICA	AESCORF	AETNAIN	AFLACINC	ALLERGA	AMERICA	APARTME	ASSURAN	AKAMAIT	ALL
2	down	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	up	stable	notrade	down	stab
3	down	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	down	stable	notrade	down	stab
4	up	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	down	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	up	stable	notrade	down	stab
5	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	down	stable	notrade	up	stab
6	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	up	stable	notrade	down	stab
7	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	down	stable	notrade	down	stab
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18	stable	stable	stable	notrade	stable	stable	stable	notrade	down	stable	up	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	down	stable	notrade	down	stab
19	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	down	stable	notrade	down	stab
20	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	stable	stable	notrade	up	stab
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22	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	down	stable	notrade	up	stab
23	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	down	stable	notrade	stable	stab
24	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	up	stable	notrade	up	stab
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28	up	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	stable	stable	notrade	down	stab
29	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	up	stable	notrade	up	stab
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31	down	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	down	stable	notrade	down	stab
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33	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	down	stable	notrade	down	stab
34	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	stable	stable	notrade	stable	stab
35	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	up	stable	notrade	down	stab
36	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	up	stable	notrade	stable	stab
37	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	down	stable	notrade	up	stab
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40	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	up	stable	notrade	down	stab
41	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	up	stable	notrade	down	stab
42	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	down	stable	notrade	down	stab
43	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	down	stable	notrade	up	stab
44	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	down	stable	notrade	stable	stab
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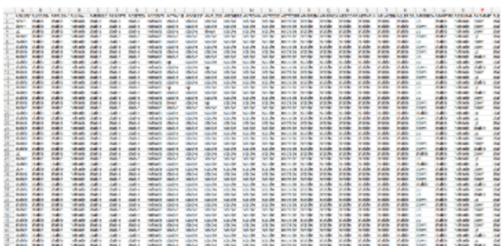
46	stable	stable	stable	notrade	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	notrade	stable	stable	stable	stable	stable	stable	down	stable	notrade	stable	stab

Portfolio management

- 500 variables
- 20 years of trading



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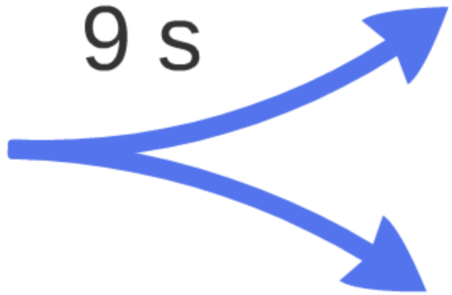
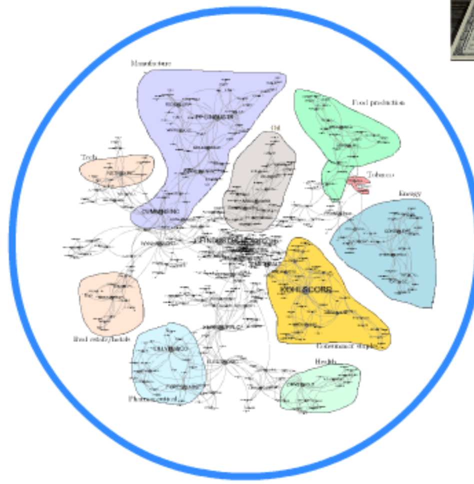


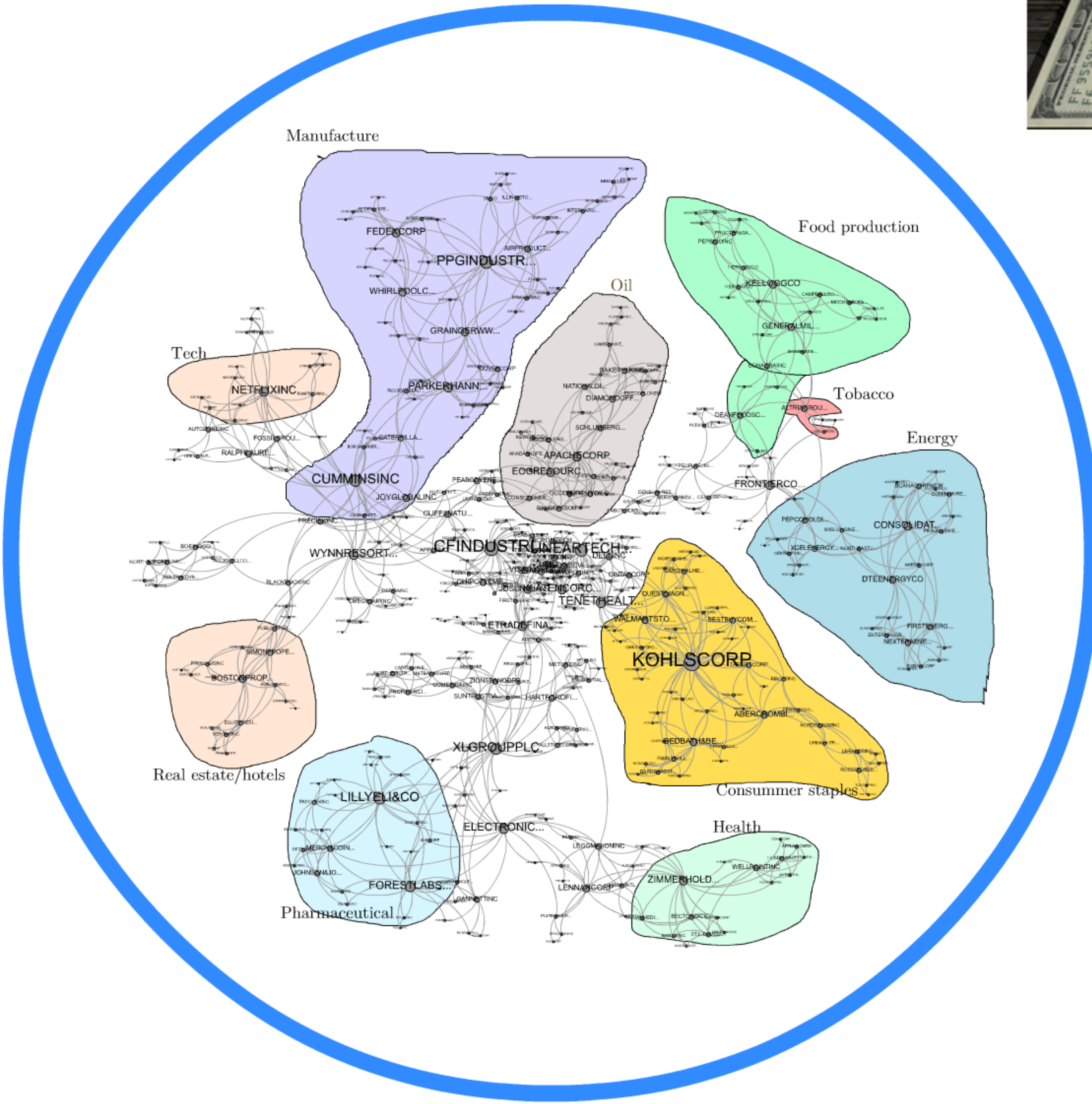


Belief propagation

Financial adviser wants to see how the market might behave given a few speculations over stocks.

evidence		NETFLIX	MERCK	RALPH LAUREN
	↘ ↗	↘ ↗	↘ ↗	↘ ↗
prior	19%-19%	23%-22%	12%-13%	23%-25%
↗	21%-21%	27%-30%	13%-13%	—
↘	21%-20%	—	34%-15%	24%-25%
↘	26%-20%	34%-26%	—	23%-25%
↗	—	—	—	25%-37%

- Netflix needs drives?
- Merck and J&J are in the same cluster
- http://www.buyupside.com/ says AMAZN and RL have 0.89 correlation coefficient
  - External factor? Sales of Ralph Lauren on Amazon.com?



Manufacture
Food production
Oil
Tech
Tobacco
Energy
Real estate/hotels
Consummer staples
Health
Pharmaceutical
FEDEXCORP
PPGINDUSTR...
WHIRLPOOLC...
GRAINGERWW...
PARKERHANN...
NETFLIXINC
CUMMINSINC
KELLOGGCO
GENERALMIL...
APACHECORP
EOGRESOURC...
FRONTIERCO...
CONSOLIDAT...
DTEENERGYCO
WYNNRESORT...
CFINDUSTR...
TENETHEALT...
KOHLSCORP
ABERCROMBI...
BOSTONPROP...
XLGROUPPLC
LILLYELI&CO
FORESTLABS...
ELECTRONIC...
ZIMMERHOLD...
LENNARCORP

Belief propagation

Financial adviser wants to see how the market might behave given a few speculations over stocks.

evidence	CAT ↘ ↗	NETFLIX ↘ ↗	MERCK ↘ ↗	RALPH LAUREN ↘ ↗
prior	19%-19%	23%-22%	12%-13%	23%-25%
Seagate ↗	21%-21%	27%-30%	13%-13%	—
Johnson&Johnson ↘	21%-20%	—	34%-15%	24%-25%
↘	26%-20%	34%-26%	—	23%-25%
a ↗	—	—	—	25%-37%

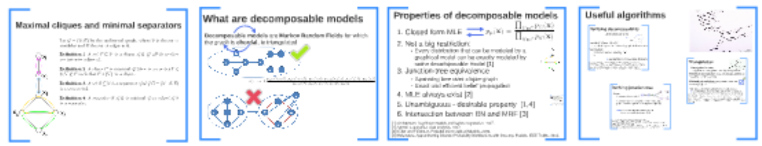
- Netflix needs drives?
- Merck and J&J are in the same cluster
- http://www.buyupside.com/ says AMAZN and RL have 0.89 correlation coefficient
  - External factor? Sales of Ralph Lauren on Amazon.com?

Scalable learning of graphical models

Introduction - Motivation

Graphical models 101

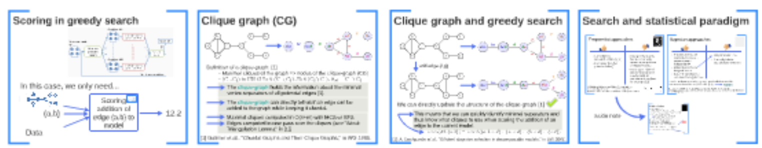
Graph theory



Evaluation - Scoring

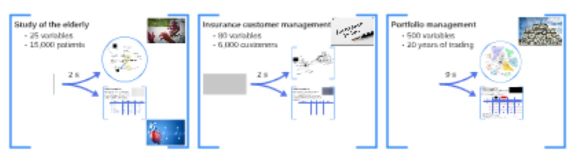
Break

Efficient search



The nitty-gritty

Use cases



Wrapping up!

This tutorial in a nutshell

1. Graphical models are extremely useful:
   - Extracting knowledge from data
   - Compact representation of high-order multivariate distributions
   - Making omnidirectional predictions
2. We can learn graphical models from datasets with 1,000+ variables https://github.com/fpetitjean/Chordalysis/
3. *Chordalysis* is the name we gave to the library that can do everything we have talked about

GitHub

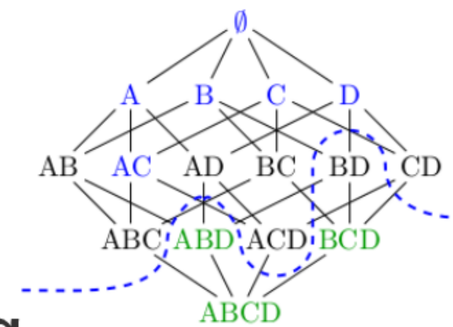
4. There is still so much work to be done!

Open problems

1. Efficient randomized search

2. Better scores (eg no Dirichlet scoring so far!)

3. Efficient storing of marginal "data"



4. Efficient data structures for counting

→ on large datasets, 99% of the CPU is used for counting

5. Learning out of core

Open problems (2)

Your community needs You!

6. How to handle numerical variables?

7. How to handle missing values?

8. Learning accurate parameters in large tables

Many problems are low-hanging fruit; you just need to pick them!

We hope that you enjoyed our tutorial on ...

Scalable learning of graphical models

François Petitjean and Geoff Webb

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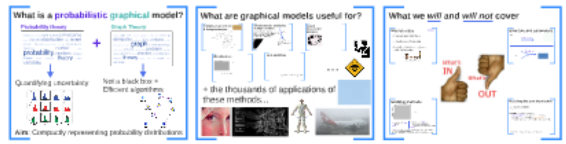
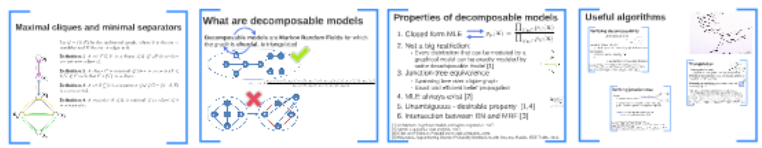
Scalable learning of graphical models

Introduction - Motivation



Graphical models 101

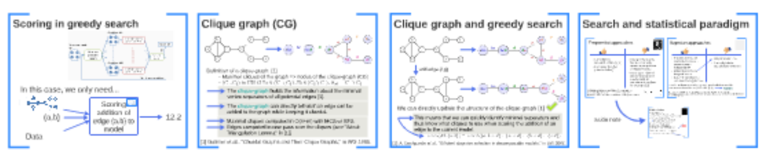
Graph theory



Evaluation - Scoring

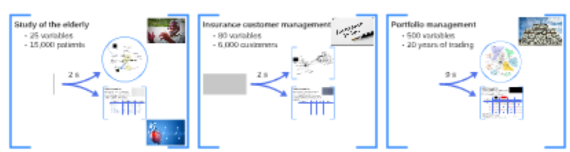
Break

Efficient search



The nitty-gritty

Use cases



Wrapping up!